GIET UNIVERSITY, GUNUPUR – 765022

Ph.D. (Second Semester) Examinations, April – 2024

PPEMT2040 – Non-Linear Functional Analysis

(Mathematics)

Time: 3 hrs Maximum: 70 Marks

The figures in the right-hand margin indicate marks.

Answer ANY FIVE Questions **(14 x 5 = 70 Marks)**

		Marks
1.a.	Prove that every closed subspaces of a complete metric space is complete.	4
b.	Let (x,d) be a complete metric space, F_n:X→X is a sequence of continuous maps. Assume that each F_n has a fixed pointx_n. If F_n→F uniformly on X. show that i. If x_n→x_0,or if Fx_0→x_0 then x_0 is a fixed point of F. ii. If F is contractive then{x_n} converges to the fixed point of F.	10
2.a.	State and prove Nadler Theorem.	14
3.a.	Let (x,d) be a complete metric space F:X → X be a map such thatF^N : X→X is contraction for N. Then for F has a unique fixed in X.	7
b.	Let (x,d) be a complete metric space and F:X→X a map, not necessarily continuous. Assume (*) for each E>0, there is a delta (E)>0 such that if d(x,Fx)<δ then F[B(x,E)≤B(x,E)Then,if d(F_u^n,F_u^{n+1})→0 for some u ∊X, then the sequences {F_u^n} converges to a fixed point for F.	7
4.a.	Let (x,d) be complete ø:X→R^+ an arbitrary non-negative function. Assume that inf {P(x)+Q(x): d(x,y)≥a}=μ(a)>0 ∀ a>0.Then each sequence {x_n}⊆X for which ϕ(x_n)→0 converges to one and the same point μ in X.	7
b.	Let (X,d) be a complete metric space. Let F: X → $\mathcal{CB}$(x) be a α contractive map. Then F has a fixed point.	7
5.a.	State and prove Bishop – Phelps theorem.	14
6.a.	Let X be a subset of a space E and f: X → E. i. X : compact, Convex ii. E: Locally convex topological vector space. iii. F = {f}: a commuting family of affine functions. Then prove that F has a fixed point.	14
7.a.	State and prove Kakutani theorem.	8
b.	The metric space of continuous function c[a,b] with the uniform metric d_∞ is complete.	6
8.a.	State and prove Banach's fixed point theorem.	
b.	Consider the initial value problem u' = 1+u^2, u(0)=0. What is the maximum of h.	4

---End of Paper---