GIET UNIVERSITY, GUNUPUR – 765022

Ph.D. (Second Semester) Examinations, November – 2023

WPPEMT2037 - Advanced Discrete Mathematics

(Mathematics)

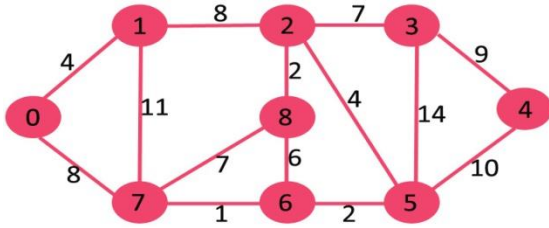Time: 3 hrs   Maximum: 70 Marks

The figures in the right hand margin indicate marks.
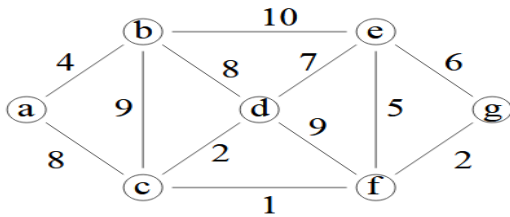
Answer ANY FIVE Questions   (14 x 5 = 70 Marks)

Marks

1.a. Show that the hypothesis "if you send me an email message, then I will finish writing the program" "if you do not send me an email message, then I will go to sleep early" and "if I go to sleep early, then I will wake up feeling refreshed" leads to a conclusion "if I do not finish writing the program then I will wake up feeling refreshed" 7

b. Show that if 'n' is an integer and n^3+5 is odd, then 'n' is even by method of contraposition. 7

2.a. Prove that $3 + 3.5 + 3.5^2 + ...+ 3.5^n = \dfrac{3(5^{n+1} - 1)}{4}$ whenever n is a non-negative integer by method of induction. 7

b. Show by method of induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for all positive integers n 7

3.a. In A survey of 100 students, it was found that 30 studied Mathematics, 54 studied Statistics, 25 studied Operations Research, 1 studied all the three subjects, 20 studied Mathematics and Statistics, 3 studied Mathematics and Operation Research and 15 studied Statistics and Operation Research. Find how many students studied none of these subjects and how many students studied only Mathematics? 7

b. Draw the Hasse diagram of the poset ({2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72}, |).
   a) Find the maximal elements.
   b) Find the minimal elements.
   c) Is there a greatest element? 7
   d) Is there a least element?
   e) Find all upper bounds of {2, 9}.
   f) Find the least upper bound of {2, 9}, if it exists.
   g) Find all lower bounds of {60, 72}.
   h) Find the greatest lower bound of {60, 72}, if it exists.

4.a. If $G = \{1, -1, i, -i\}$ and $H = \{1, -1\}$ be a sub-group of G under the operation multiplication then find all the left cosets of H in G. 7

b. Let G be a group congaing even number of elements, then prove that there exist at least one element which has its own inverse. 7

5.a. Show that if meet operation is distributive over join operation then join operation distributive over meet operation. 7

b. Let $E(x_1, x_2, x_3) = \overline{\overline{(x_1 \vee x_2)} \vee (\overline{x_1} \wedge x_3)}$ be a Boolean expression. Find its disjunctive and conjunctive normal forms. 7

6.a. find the shortest path from the vertex 0 to the vertex 4 by using Dijkstra's algorithm   7

b. Find the minimum spanning tree of the following graph by using Prim's's algorithm.   7

7 a. Prove that in a distributive lattice if an element has a complement then this complement is unique.   7

b. For any a, b.c and d in a lattice $(A, \leq)$ if $a \leq b$ and $c \leq d$ then show that $a \vee c \leq b \vee d$ and $a \wedge c \leq b \wedge d$.   7

8 a. State and prove demorgan's property of distributive lattice.   7

b. Let $(A, \leq)$ be a distributive lattice. show if $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ for some a then show that $x = y$.   7

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