QP Code: RM22MSC119 Reg. No | | | | | | | | | | | AY 22

GIET UNIVERSITY, GUNUPUR – 765022

M. Sc. (Fourth Semester) Examinations, May – 2024

20MTPC403 – Functional Analysis - II

(Mathematics)

Time: 3 hrs Maximum: 70 Marks

(The figures in the right hand margin indicate marks.)

PART – A **(2 x 10 =20 Marks)**

Q.1	Answer ALL Questions	CO#	Blooms Level
a.	Define unitary and normal operator	CO1	K1
b.	State banach –alaoglu theorem.	CO2	K2
c.	Define weak*convergent	CO2	K2
d.	Define bounder operator	CO2	K2
e.	Define bounded below.	CO2	K2
f.	state the Riesz-Fischer theorem	CO2	K2
g.	State the Polarization identity	CO3	K1
h.	Define separable set.	CO4	K2
i.	Define weak convergent.	CO4	K2
j.	State the Bolzano-Weierstrass theorem.	CO4	K2

PART – B **(10 x 5=50 Marks)**

	<u>Answer ***ANY FIVE*** the questions</u>	Marks	CO#	Blooms Level
2.	State and prove unique Hahn-Banach extension theorem.	10	C01	K3
3.	State and prove projection theorem.	10	C01	K3
4.	State and prove Riesz representation theorem	10	C02	K3
5.	Let H be a Hilbert space then prove, Let A and B be self-adjoint. Then A+B is self adjoint. Also AB is self –adjoint if and only if A and B commute	10	C03	K3
6.	Let H be a Hilbert space and A$\in BL(H)$ then prove that (a) Z(A)=R$(A^*)^\perp$ and Z(A*)=R$(A\ )^\perp$ A is injective if and only if R(A*) is dense in H, and A* is injective if and only if R(A) is dense in H (b)R(a)=H if and only if A* is bounded below, and R(A*)=H if and only if is A is bounded below	10	C03	K3
7.	Let X be a normed space and (x_n') be a sequence in X' prove that if (x_n') is bounded and (x_n'(x)) is a Cauchy-sequence in K for each x in a subset of X whose span is dense in X, then (x_n') is weak convergent in X'.the converse holds if X is a Banach space.	10	C04	K3
8.	Let H be a Hilbert space .consider A,B$\in BL(H) AND\ k \in k. Then\ prove\ that$ (A+B)*=A*+B*,(KA*)=$\bar{k}$A*,(AB)*=B*A*,(A*)*=A. Further, A is invertible if and only if A* is invertible	10	C04	K3