QP Code: RD22MSC119 Reg. No | | | | | | | | | | | AY 22

GIET UNIVERSITY, GUNUPUR – 765022

M. Sc. (Third Semester) Regular Examinations, December – 2023

22MTPC301 - Measure Theory & Integration

(Mathematics)

Time: 3 hrs Maximum: 70 Marks

(The figures in the right hand margin indicate marks.)

PART – A **(2 x 10 = 20 Marks)**

Q.1. Answer ***ALL*** questions	CO #	Blooms Level
a. Define equivalence relation with example.	CO1	K1
b. Define outer measure.	CO1	K1
c. Explain Every subset of a countable set is countable.	CO1	K2
d. Prove that ϕ is measurable set.	CO2	K3
e. Define step function.	CO2	K3
f. Define Monotone Convergence Theorem.	CO2	K2
g. Explain Differentiation of Monotone functions.	CO3	K1
h. Explain absolutely continuous functions.	CO3	K2
i. State Minkowski Inequality Theorem.	CO4	K1
j. Define Banach Spaces.	CO4	K2

PART – B **(10 x 5 = 50 Marks)**

Answer ***ANY FIVE*** questions	Marks	CO #	Blooms Level
2. a. Prove that the union of a finite collection of measurable sets is measurable.	5	CO1	K1
b. Prove that The outer measure of an internal is its length.	5	CO1	K1
3.a Let A be any set and $\langle E_i \rangle$ be a sequence of disjoint measurable sets and. Then $m^*(A \cap \bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} m^*(A \cap E_i)$.	7	CO1	K1
b Prove that If $m^*A = 0$, $then\ m^*(A \cup B) = m^*B$.	3	CO1	K2
4.a Lebesgue measure is countably additive, that is, if $\{E_k\}_{k=1}^{\infty}$ is a countable disjoint collection of measurable sets, then its union $U_{k=1}^{\infty} E_k$ also is measurable and $m(U_{k=1}^{\infty} E_k) = \sum_{k=1}^{\infty} m(E_k)$.	5	CO2	K2
b Prove that the function *f has* a measurable domain E. Then the following statements are equivalent: i. For each real number α, the set $\{x : f(x) > \alpha\}$ is measurable. ii. For each real number α, the set $\{x : f(x) \geq \alpha\}$ is measurable. iii. For each real number α, the set $\{x : f(x) < \alpha\}$ is measurable. iv. For each real number α, the set $\{x : f(x) \leq \alpha\}$is measurable. Each of these properties implies that for each extended real number c, the set $\{x : f(x) = \alpha\}$ is measurable.	5	CO2	K2

5.a	Let f *and* g be integrable over E. Then: i. The function αf is integrable over E, and $\int_E' \alpha f = \alpha \int_E' f$. ii. Then function $f + g$ is integrable over E, and $$\int_E' f + g = \int_E' f + \int_E' g \ .$$ iii. $If\ \ f \le g\ \ then\ \int_E^{\cdot} f \le \int_E^{\cdot} g$ iv. If A and B are disjoin measurable sets contained in E, then $$\int_{A \cup B}^{\cdot} f = \int_A^{\cdot} f + \int_B^{\cdot} f$$	5	CO2	K2
b	Show that if $f(x) = \begin{cases} 0, & if\ x\ is\ \ a\ irrational \\ 1, if & x\ is\ a\ rational, \end{cases}$ Then $\mathrm{R}\int_a^{\bar{b}} f(x)dx = b - a$ and $\mathrm{R}\int_{\underset{\smile}{a}}^{b} f(x)dx = 0$.	5	CO2	
6.a	If $\langle f_n \rangle$ is a sequence of nonnegative measurable function and f_n(x)→ $f(x)$ almost every where on a set E, then $\int_E^{\cdot} f \le \ \underline{lim} \int_E^{\cdot} f_n$	5	CO3	K2
b	Let f be defined and bounded on a measurable set E with mE finite. In order that $\inf_{f \le \psi} \int_E^{\cdot} \psi(x)\, dx = \sup_{f \ge \varphi} \int_E^{\cdot} \varphi(x)\, dx$ For all simple function $\varphi$and ψ, I is necessary and sufficient that *f* be measurable	5	CO3	
7.	If the function *f* is monotone on the open interval [a, b]. Then *f* is differentiable almost everywhere. The derivative *f*′ is measurable, and $$\int_a^b f(x)\, dx \le f(a) - f(b)$$	10	CO3	K3
8.a	If p *and* q are nonnegative extended real number such that $\frac{1}{p} + \frac{1}{q} = 1$ and If $f \in L^p\ and\ g \in L^p, then\ f.g\ \in L^1\ and$ $$\int \lvert f.g \rvert = \lVert f \rVert_p . \lVert g \rVert_p$$ Equality holds if and only if for some constant $\alpha\ and\ \beta$, not both are zero, we have $\alpha \lvert f \rvert^p = \ \beta \lvert g \rvert^q$	5	CO4	K3
b	Let f *and* g are in $\mathrm{L}^{\mathrm{P}} L^p\ \ with\ 1 \le p \le \infty,$ then so dose their sum $f + g$ and $\lVert f + g \rVert_p = \lVert f \rVert_p + \lVert g \rVert_p$ If $1 < p < \infty$, then equality can hold only if there are nonnegative constants $\alpha\ and\ \beta$ such that $\beta f = \alpha g$.	5	CO4	K3

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