QP Code: RJ22MSC083 Reg. No | | | | | | | | | | | AR 22

GIET UNIVERSITY, GUNUPUR – 765022

M. Sc (Second Semester) Examinations, July – 2023

22MTPC203 - Partial Differential Equation

(Mathematics)

Time: 3 hrs Maximum: 70 Marks

(The figures in the right hand margin indicate marks.)

PART – A **(2 x 10 = 20 Marks)**

Q.1. Answer ***ALL*** questions	CO #	Blooms Level
a. Formulate the PDE of the surface $z = xy + F(x^2 + y^2)$.	CO1	K2
b. Solve $x^2 p + y^2 q = z^2$	CO1	K2
c. Solve $(p - q)(z - px - qy) = 1$	CO1	K2
d. Solve $zpq = p + q$.	CO2	K2
e. Classification of the partial differential $U_{xx} - 2\sin x\, U_{xy} - coc^2 x\, U_{yy} - \cos x U_y = 0$	CO2	K3
f. Define hyperbolic Partial Differential equation with an example.	CO2	K2
g. State the Interior Dirichlet problem for a circle.	CO3	K1
h. Define Dirac delta function and write any two properties.	CO3	K1
i. Define Poisson's equation	CO4	K1
j. Solve Laplace equation $u_{xx} + u_{yy} = 0$ by Separating Variables method.	CO4	K2

PART – B **(10 x 5 = 50 Marks)**

<u>Answer ***ANY FIVE*** questions</u>	Marks	CO #	Blooms Level
2. a. Formulate the PDE $xyz = f(x + y + z)$.	5	CO2	K1
b. Find the integral domain surface of the following PDE $(x - y)y^2 p + (y - x)x^2 q = (x^2 + y^2)z$ passes through $y = 0,\ xz = a^3$	5	CO2	K1
3. Find the characteristics of the PDE $p^2 + q^2 = 2$ and determine the integral surface which passes through $x = 0,\ z = y$.	10	CO2	K1
4. Reduce to canonical form and find the general solution of $y^2 U_{xx} - 2xyU_{xy} + x^2 U_{yy} = \frac{y^2}{x} Ux + \frac{x^2}{y} U_y$	10	CO3	K2
5. Derive the interior Dirichlet problem for a circle. $\nabla^2 u = 0, \quad 0 \le r \le a, \quad 0 \le \theta \le 2\pi$ BCs: $u(a, \theta) = f(\theta),, \quad 0 \le \theta \le 2\pi$	10	CO3	K2
6. Solve one-dimensional wave equation by canonical reduction.	10	CO2	K2

7.	A stretched string of finite length L is held fixed at its ends and is subjected to an initial displacement $u(x, 0) = u_0 \sin(\pi x/L)$. The string is released from this position with zero initial velocity. Find the resultant time dependent motion of the string.	10	CO3	K3
8.	A string of length L is released from rest in position $y = f(x)$. Show that the total energy of string is $\frac{\pi^2 T}{4L}\sum_{n=1}^{\infty} s^2 K_s^2$ Where $K_s = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{s\pi x}{L}\right)dx$ and total energy $E = \frac{T}{2}\int_0^L \left[\left(\frac{\partial y}{\partial x}\right)^2 + \frac{1}{c^2}\left(\frac{\partial y}{\partial t}\right)^2\right]dx.$ T is tension of the string. If the mid- point of a string is pulled aside through a small distance and then released. Show that in the subsequent motion the fundamental mode contributes $8/\pi^2$ of the total energy	10	CO4	K3

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