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GIET UNIVERSITY, GUNUPUR - 765022
M. Sc (Second Semester) Regular Examinations, July - 2023
22MTPC201- Linear Algebra
(Mathematics)

Time: 3 hrs

Maximum: 70 Marks

(The figures in the right hand margin indicate marks.)

PART – A**(2 x 10 = 20 Marks)**Q.1. Answer **ALL** questions

	CO #	Blooms Level
a. Define inner product space and give an example.	1	1
b. Define dual space with example	1	1
c. What is minimal polynomial? Give an example.	2	1
d. Write the statement of the Rank-Nullity theorem?	2	1
e. Write down the step's to find the solution of system of linear equations in Cramer's rule.	2	1
f. What do you mean by LU-decomposition of a square matrix?	3	1
g. Prove that the eigen values of Hermitian matrix A are real.	3	2
h. Show that A^{-1} exists iff 0 is not an eigen value of A .	4	2
i. Define eigenvalue. Write down any matrix and find it's eigen values.	4	2
j. Write a matrix and verify the Caley-Hamilton Theorem.	4	2

PART – B**(10 x 5 = 50 Marks)**Answer **ANY FIVE** questions

	Marks	CO #	Blooms Level
2. a. What is Gram-Schmidt Orthogonalization Process?	2	1	3
b. Write the derivation for Gram-Schmidt Orthogonalization Process.	8	1	2
3.a. If V and W are of dimensions m and n respectively over F then $\text{Hom}(V, W)$ is of dimension mn over F .	10	1	2
4. a. What do you mean by basis of a vector space?	2	2	1
b. Let $D: P_3 \rightarrow P_2$ be defined $D(p) = p'$. Let $B_1 = \{1, x, x^2, x^3\}$ be the standard basis for P_3 . Let $B_2 = \{1, x, x^2\}$ be the standard basis for P_2 . Find matrix of D relative to B_1 and B_2 .	8	2	3
5.a. If $T \in A(V)$ has all its characteristic roots in F , then there is a basis of V in which the matrix of T is triangular.	10	2	2
6. a. Assume that $p(t)$ is a minimal polynomial of a linear operator T on a finite dimensional vector space V . Show that if $g(T) = 0$, then $p(t)$ divides $g(t)$, for any polynomial $g(t)$ then the minimal polynomial $p(t)$ divides the characteristic polynomial of T .	6	3	3

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| b. | Find the minimal polynomial of the matrix $A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ 1 & -1 & 2 \end{bmatrix}$. | 4 | 3 | 3 |
| 7.a. | What do you mean by diagonalization of a matrix? | 2 | 4 | 1 |
| b. | Find the modal matrix and diagonalizes the matrix $C = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$. | 8 | 4 | 3 |
| 8. a. | Determine the nature, index and signature of the quadratic form $2x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 - 4x_1x_3 - 4x_2x_3$. | 6 | 4 | 3 |
| b. | Show that Transpose of a unitary matrix is unitary. | 4 | 4 | 2 |

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