GIET UNIVERSITY, GUNUPUR – 765022

M. Sc. (First Semester) Regular Examinations, February – 2024

22MTPC102 – Topology

(Mathematics)

Time: 3 hrs Maximum: 70 Marks

(The figures in the right hand margin indicate marks.)

PART – A **(2 x 10 = 20 Marks)**

Q.1. Answer ***ALL*** questions

		CO #	Blooms Level
a.	Define discrete topology and give one example.	CO1	K1
b.	Define Closure of topology with example.	CO1	K1
c.	Explain Relative topology.	CO1	K2
d.	Show that A^0 is largest open subset of A.	CO2	K2
e.	Define compactness and give one example.	CO2	K1
f.	Show that C = $\left\{\left(0,\frac{n}{n+1}\right) : n \in N\right\}$ is a cover of (0,1).	CO2	K3
g.	Define connected and disconnected space.	CO3	K1
h.	Define regular space.	CO3	K1
i.	Show that every discrete space is a Hausdorff space.	CO4	K2
j.	Define metric space.	CO4	K2

PART – B **(10 x 5 = 50 Marks)**

<u>Answer ***ANY FIVE*** questions</u>

			Marks	CO #	Blooms Level
2.	a.	Let (X,T) be a topological space and let A, B are non empty subsets of X, then prove that i. $D(\emptyset) = \emptyset$ ii. $x \in D(A) \Longrightarrow x \in D(A - \{x\})$ iii. $A \subset B \Longrightarrow D(A) \subset D(B)$ IV. $D(A \cup B) \Longrightarrow D(A) \cup D(B)$ V. $D(A \cap B) \Longrightarrow D(A) \cap D(B)$	5	CO1	K3
	b.	Let (X,T) be a topological space and A⊂X. A point x of A is an interior point of A iff it is not a limit point of X - A.	5	CO1	K2
3.	a.	Let T ={X ,∅, {p},{p, q},{p, q, t},{p ,q ,r ,s},{p, r, s}} be the topology on X={p, q, r, s, t}.Determine limit points, closure, interior, exterior and boundary of the following sets : i. A = {r, s, t} ii. B = {q}	5	CO1	K2
	b.	If T_1, T_2 are two topologies defined on the same set X, then $T_1 \cap T_2$ is also a topology on X, but $T_1 \cup T_2$ is not a topology on X.	5	CO1	K2

4. a.	Show that a closed subset of a compact space is compact	5	CO2	K3
b.	If A, B be separated sets of a topological space (X,T) then prove that i. $A \cup B$ *is closed* $\Longrightarrow A$ *and* B *are closed* ii. $A \cup B$ *is open* $\Longrightarrow A$ *and* B *are open.*	5	CO2	K2
5.a.	Prove that the union of two non-empty separated subset of a topological space is disconnected.	5	CO2	K2
b.	If **f** and **g** are continuous real or complex functions defined on a topological space **X,** then $f + g$, αf *and* fg are also continuous . Furthermore , if f *and* g are real then $f \vee g$ *and* $f \wedge g$, *are continuous.*	5	CO2	K3
6. a.	A topological space (X, T) is a T_0-space iff for any distinct arbitrary points x, y of X, the closure of {x} and {y} are distinct	5	CO3	K3
b.	A topological space (X, T) is a T_1-space *iff* T contains the cofinite topology on X	5	CO3	K2
7.a.	Show that (X, T) is a T_4-space $\Longrightarrow$ (X, T) is a T_2-space.	5	CO3	K2
b.	Prove that the T_1-axiom of separation is a hereditary property.	5	CO3	K2
8. a.	State and prove Holder's inequality.	5	CO4	K2
b.	State and prove Hilbert space.	5	CO4	K2