GIET UNIVERSITY, GUNUPUR – 765022

M. Sc. (First Semester) Regular Examinations, February-2024

22MTPC104 - Integral Transforms

(Mathematics)

Time: 3 hrs. Maximum: 70 Marks

(The figures in the right-hand margin indicate marks.)

PART – A **(2 x 10 = 20 Marks)**

		CO#	Blooms Level
Q1.	Answer ***ALL*** questions		
a.	$L\{e^{ax+b}\} =$ ____________	CO1	K1
b.	$\int_0^\infty e^{-x^2}dx =$ _________	CO1	K2
c.	Find the inverse Laplace transform of $\frac{1}{\sqrt{s}}$	CO1	K1
d.	Define an odd function with an example.	CO2	K1
e.	Write the Fourier integral formula of a function f(x).	CO2	K1
f.	If the function $f(x) = x^3 + 10$ is defined in the interval $(-\pi, \pi)$ then find the Fourier series coefficient a_0	CO2	K1
g.	Evaluate the Z transforms of n^2.	CO3	K1
h.	Evaluate $Z(5^n)$	CO3	K1
i.	$Z^{-1}(e^z) =$ ________	CO3	K1
j.	Define Dirac's delta function (or unit impulse) function.	CO4	K1

PART – B **(10 x 5 = 50 Marks)**

	Answer ***ANY FIVE*** questions	Marks	CO#	Blooms Level
2. a.	If $L\{f(t)\} = f(s)$ then prove that $L\{f(at)\} = \frac{1}{a}f(\frac{s}{a})$.	5	CO1	K2
b.	Let f(s) denote the Laplace transform of a function f(t) then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty f(s)ds$.	5	CO1	K2
3. a.	If $L^{-1}\{f(s)\} = f(t)$ then prove that $L^{-1}\{e^{-as}f(s)\} = g(t)$, Where $g(t) = \begin{cases} f(t-a), t > a \\ 0, t < a \end{cases}$.	5	CO1	K2
b.	Find the inverse Laplace transform of $\frac{s}{2s^2-8}$.	5	CO1	K2
4. a.	Find the Fourier series of f(x) = $\begin{cases} 1, if -1 < x < 0 \\ -1, if\ 0 < x < 1 \end{cases}$.	5	CO2	K3
b.	State and proof of the linearity property of the Fourier transform	5	CO2	K2
5.a.	Express $f(x) = \begin{cases} 1, & if\ 0 < x < \pi \\ 0, & if\ \ x > \pi \end{cases}$ as a Fourier sine integral.	5	CO2	K3
b.	Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & if\ \|x\| < 1 \\ 0, & if\ \|x\| > 1 \end{cases}$	5	CO2	K2
6. a.	If $L^{-1}\{f(s)\} = f(t)$ then prove that$L^{-1}\left\{\frac{d^n}{ds^n}f(s)\right\} = (-1)^n t^n f(t)$.	5	CO3	K2
b.	Find the inverse Z Laplace transforms of $\frac{3z}{(z-1)(z+2)}$.	5	CO3	K2
7.a.	Solve the difference equation $y_{k+2} - y_{k+1} + y_k = 3^k$ with $y_0 = 0, y_1 = 1$.	5	CO3	K2

P.T.O

	b. Evaluate $Z(e^{at}\cos bt)$	5	CO3	K2
8.	a. Evaluate $\int_0^a rJ_0(\xi r)dr$	5	CO4	K2
	b. Obtain the solution of $u_{tt} = c^2\left(u_{rr} + \frac{1}{r}u_r\right), 0 < r < \infty,\ t > 0,\ u(r,\ 0) = f(r), u_t(r,\ 0) = g(r)$ where c is a constant.	5	CO4	K3

--- End of Paper ---