QPC: RF23MSC005 Reg. No | | | | | | | | | | AY 23

GIET UNIVERSITY, GUNUPUR – 765022

M. Sc. (First Semester) Examinations, February – 2024

22MTPC101 - Abstract Algebra

(Mathematics)

Time: 3 hrs Maximum: 70 Marks

(The figures in the right hand margin indicate marks.)

PART – A **(2 x 10 = 20 Marks)**

Q. 1.	Answer ***ALL*** questions	CO#	Blooms Level
a.	List all the elements of Z_{40} of order 10.	CO1	K2
b.	Create a subgroup from the set $G = \{1, -1, i, -i\}$.	CO1	K2
c.	Let G be the group of positive real numbers under multiplication and $\overline{G}$ be the group of all real number under addition. A mapping $\phi: G \to \overline{G}$ is defined by $\phi(x) = \log_{10} x$ for all $x \in G$. Then show that ϕ is a homomorphism.	CO1	K2
d.	$f: Z_8 \to Z_8$. How many isomorphism possible?	CO2	K1
e.	Is the set $\{(1,1,0), (1,0,1), (0,1,1)\}$, a basis for V_3?	CO3	K2
f.	What is Unique Factorization Domain?	CO3	K1
g.	Define Einstein Criterion and justify your answer with example.	CO3	K1
h.	Is the set of real numbers is a field? If not then justify your answer.	CO2	K1
i.	What is the difference between algebraic number and algebraic extension? Give an example.	CO4	K2
j.	What do you mean by extension field? Justify your answer with an example.	CO4	K2

PART – B **(10 x 5 = 50 Marks)**

	Answer ***ANY FIVE*** questions	Marks	CO#	Blooms Level
2.	Let ϕ be a homomorphism of G onto $\bar{G}$ with kernel K. Then $\frac{G}{K} \approx \bar{G}$.	10	CO1	K2
3. a.	Show that $G = \{1,3,5,7\}$ is a group under multiplication modulo 8.	5	CO1	K2
b.	How many elements of order 2 are present in $D_4 \times D_4$?	5	CO1	K2
4.	Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then R is a field.	10	CO2	K2
5. a.	Prove that normalizer of a in G, defined by $N(a)$ is a subgroup of G.	6	CO2	K2
b.	The characteristic of an integral domain is either 0 or a prime number.	4	CO2	K2
6. a.	If U and W are two subspaces of a vector space V, prove that U and $U + W = U$ iff $U \subset W$.	6	CO3	K2
b.	Let V be any vector space. Then the set $\{v\}$ is L.D. iff $v = \vec{0}$.	4	CO3	K1
7.a.	In an n-dimensional vector space V, any set of n L.I. vectors is a basis.	5	CO3	K2
b.	If S is a non-empty subset of a vector space V. Prove that $[S]$, is the intersection of all subspaces of V containing S.	5	CO3	K2
8.	Let $K\|F$ be any field extension. Then, $a \in K$ is algebraic over F if and only if $[F(a): F]$ is finite, i.e. $F(a)$ is a finite extension over F. Moreover $[F(a): F] = n$, when n is the degree of minimal polynomial of a over F.	10	CO4	K2

--- End of Paper ---