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**GIET UNIVERSITY, GUNUPUR – 765022**  
M. Sc. (First Semester) Examinations, March – 2023  
**22MTPC104 – Integral Transformation**  
(Mathematics)

Time: 3 hrs.

Maximum: 70 Marks

(The figures in the right-hand margin indicate marks.)

**PART – A****(2 x 10 = 20 Marks)**Answer ALL questions

- |  | CO# | Blooms<br>Level |
|--|-----|-----------------|
| a. $L\{e^{ax+b}\} = \underline{\hspace{2cm}}$  | 1   | K1              |
| b. $\int_0^\infty e^{-x^2} dx = \underline{\hspace{2cm}}$  | 1   | K2              |
| c. Find the inverse Laplace transform of $\frac{1}{\sqrt{s}}$  | 1   | K1              |
| d. Define an odd function with an example.   | 2   | K1              |
| e. Write the Fourier integral formula of a function f(x).  | 2   | K1              |
| f. If the function $f(x) = x^3 + 10$ is defined in the interval $(-\pi, \pi)$ then find the Fourier series coefficient $a_0$ | 2   | K1              |
| g. Evaluate the Z transforms of $n^2$ .  | 3   | K1              |
| h. Evaluate $Z(5^n)$   | 3   | K1              |
| i. $Z^{-1}(e^z) = \underline{\hspace{2cm}}$  | 3   | K1              |
| j. Define Dirac's delta function (or unit impulse) function.   | 4   | K1              |

**PART – B****(10 x 5 = 50 Marks)**Answer ANY FIVE questions

- |  | Marks | CO<br># | Blooms<br>Level |
|--|-------|---------|-----------------|
| 2. a. If $L\{f(t)\} = f(s)$ then prove that $L\{f(at)\} = \frac{1}{a}f\left(\frac{s}{a}\right)$ .  | 5     | 1       | K2              |
| b. Let f(s) denote the Laplace transform of a function f(t) then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty f(s)ds$ .                  | 5     | 1       | K2              |
| 3. a. If $L^{-1}\{f(s)\} = f(t)$ then prove that $L^{-1}\{e^{-as}f(s)\} = g(t)$ , Where $g(t) = \begin{cases} f(t-a), t > a \\ 0, t < a \end{cases}$ . | 5     | 1       | K2              |
| b. Find the inverse Laplace transform of $\frac{s}{2s^2-8}$ .  | 5     | 1       | K2              |
| 4. a. Find the Fourier series of $f(x) = \begin{cases} 1, & \text{if } -1 < x < 0 \\ -1, & \text{if } 0 < x < 1 \end{cases}$                           | 5     | 2       | K3              |
| b. State and proof of the linearity property of the Fourier transform  | 5     | 2       | K2              |
| 5. a. Express $f(x) = \begin{cases} 1, & \text{if } 0 < x < \pi \\ 0, & \text{if } x > \pi \end{cases}$ as a Fourier sine integral.                    | 5     | 2       | K3              |
| b. Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & \text{if }  x  < 1 \\ 0, & \text{if }  x  > 1 \end{cases}$                             | 5     | 2       | K2              |
| 6. a. If $L^{-1}\{f(s)\} = f(t)$ then prove that $L^{-1}\left\{\frac{d^n}{ds^n}f(s)\right\} = (-1)^n t^n f(t)$ .                                       | 5     | 3       | K2              |
| b. Find the inverse Z Laplace transforms of $\frac{3z}{(z-1)(z+2)}$ .  | 5     | 3       | K2              |
| 7. a. Solve the difference equation $y_{k+2} - y_{k+1} + y_k = 3^k$ with $y_0 = 0, y_1 = 1$ .  | 5     | 3       | K2              |
| b. Evaluate $Z(e^{at} \cos bt)$  | 5     | 3       | K2              |
| 8. a. Evaluate $\int_0^a r J_0(\xi r) dr$  | 5     | 4       | K2              |
| b. Solve $u_{k+2} - 2u_{k+1} + u_k = 3n + 5$ .   | 5     | 4       | K3              |

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