QP Code: RM22MSC007 Reg. No | | | | | | | | | | AR 22

# GIET UNIVERSITY, GUNUPUR – 765022

M. Sc. (First Semester) Examinations, March – 2023

## 22MTPC101 - Abstract Algebra

(Mathematics)

Time: 3 hrs Maximum: 70 Marks

**(The figures in the right hand margin indicate marks.)**

**PART – A** **(2 x 10 = 20 Marks)**

| Q. 1. Answer ***ALL*** questions | | CO# | Blooms Level |
|---|---|---|---|
| a. | Let $G = \{1, \omega, \omega^2\}$ is a group. Find the order of each element of the group. | 1 | K2 |
| b. | Create a subgroup from the set $G = \{1, -1, i, -i\}$. | 1 | K2 |
| c. | Let $G$ be the group of positive real numbers under multiplication and $\overline{G}$ be the group of all real number under addition. A mapping $\phi: G \to \overline{G}$ is defined by $\phi(x) = \log_{10} x$ for all $x \in G$. Then show that $\phi$ is a homomorphism. | 1 | K2 |
| d. | Is the set of integers is a cyclic group w.r.t addition? If yes then find its generator. | 1 | K1 |
| e. | Is the set $\{(1,1,0), (1,0,1), (0,1,1)\}$, a basis for $V_3$? | 3 | K2 |
| f. | What is Unique Factorization Domain? | 3 | K1 |
| g. | Define Einstein Criterion. | 3 | K1 |
| h. | Write an example for field with proper explanation. | 2 | K1 |
| i. | What is the difference between algebraic number and algebraic extension? Give an example. | 4 | K2 |
| j. | What do you mean by extension field? Justify your answer with an example. | 4 | K2 |

**PART – B** **(10 x 5 = 50 Marks)**

| Answer ***ANY FIVE*** questions | | Marks | CO # | Blooms Level |
|---|---|---|---|---|
| 2. a. | Let $\phi$ be a homomorphism of $G$ onto $\bar{G}$ with kernel $K$. Then $\frac{G}{K} \approx \bar{G}$. | 10 | 1 | K2 |
| 3.a. | Show that $G = \{1,3,5,7\}$ is a group under multiplication modulo 8. | 5 | 1 | K2 |
| b. | If $G$ is a group, such that $(a.b)^2 = a^2.b^2$ for all $a, b \in G$. Show that $G$ must be abelian. | 5 | 1 | K2 |
| 4. a. | Let $R$ be a commutative ring with unit element whose only ideals are $(0)$ and $R$ itself. Then $R$ is a field. | 10 | 2 | K2 |
| 5.a. | Prove that normalizer of $a$ in $G$, defined by $N(a)$ is a subgroup of $G$. | 6 | 2 | K2 |
| b. | If R is a ring, then for all $a, b \in$ R show that, $a(-b) = (-a)b = -(ab)$. | 4 | 2 | K2 |
| 6. a. | If $U$ and $W$ are two subspaces of a vector space $V$, prove that $U$ and $U + W = U$ iff $U \subset W$. | 6 | 3 | K2 |
| b. | Let $V$ be any vector space. Then the set $\{v\}$ is linearly dependent L.D. iff $v = \vec{0}$. | 4 | 3 | K1 |
| 7.a. | In an $n$-dimensional vector space $V$, any set of $n$ linearly independent vectors is a basis. | 5 | 3 | K2 |
| b. | If $S$ is a non-empty subset of a vector space $V$. Prove that $[S]$, is the intersection of all subspaces of $V$ containing $S$. | 5 | 3 | K2 |
| 8. a. | Let $K\|F$ be any field extension. Then, $a \in K$ is algebraic over $F$ if and only if $[F(a): F]$ is finite, i.e. $F(a)$ is a finite extension over $F$. Moreover $[F(a): F] = n$, when $n$ is the degree of minimal polynomial of $a$ over $F$. | 10 | 4 | K2 |

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