GIET UNIVERSITY, GUNUPUR – 765022

M. Sc. (Third Semester) Examinations, December – 2022

20MTPC301 - FUNCTIONAL ANALYSIS-1

(MATHEMATICS)

Time: 3 hrs Maximum: 70 Marks

(The figures in the right hand margin indicate marks.)

PART – A **(2 x 10 = 20)**

Q.1. Answer ALL Questions		CO#	Blooms Level
a.	Define normed space with an example?	CO1	K_1
b.	What do we mean by a convex set?	CO1	K_1
c.	Define open ball and closed ball.	CO1	K_1
d.	What is the condition for a mapping $F: X \to Y$ to be a Homeomorphism?	CO2	K_1
e.	Define the Banach space.	CO2	K_1
f.	State the Baire's theorem.	CO2	K_1
g.	Define the interior point and the limit point of a space E.	CO3	K_1
h.	State the bounded inverse theorem	CO3	K_1
i.	Define spectral radius.	CO4	K_1
j.	State the Banach-Steinhaus theorem.	CO4	K_1

PART – B **(10 x 5 = 50 Marks)**

<u>Answer ***ANY FIVE*** questions</u>		Marks	CO#	Blooms Level
2.	Let X be a normed space. Then prove that the following conditions are equivalent. i. Every closed and bounded subset of X is compact. ii. The subset $x \in X: \|\|x\|\| \leq 1$ of X is compact. $iii.$ X is finite dimensional	10	CO1	K_3
3	Let X and Y be normed spaces and $F: X \to Y$ be a linear map such that the range $R(F)$ of F is finte dimensional. Then show that, if F is continuous if and only if the zero space $Z(F)$ of F is closed in X	10	CO1	K_2
4.	State and prove Hahn-Banach extension theorem	10	CO2	K_2
5.	Prove that, a normed space X is a Banach space if and only if every absolutely summable series of elements in X is summablein X.	10	CO2	K_3
6.	State and prove the closed graph theorem	10	CO3	K_3
7.	State and proof the uniform boundedness principle	10	CO3	K_3
8.	Let X be a normed space, and $A \in BL(X)$ be of finite rank. Then prove that $\sigma_e(A) = \sigma_a(A) = \sigma(A)$.	10	CO4	K_3

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