Set - 1

M. Tech (Structural Engg.)

MSEPC 1030 - Matrix Method of Analysis of Structure

Answer key.

PART. A.

a. Degree of redundancy = 1

b. Kinematic indeterminacy = 2.

c. It help us to represent forces and displacements at the element level. corresponding to every node in the overall structure, there will be as many sets of local co-ordinates as there are are elements joining at the node under consideration.

d. ~~Any~~ The matrix which transforms any desired information of the structure to its corresponding element information or vice versa is termed as transformation matrix.

e. $f = \frac{l}{AE}$

f. The element ~~of~~ stiffness matrix is the inverse of element flexibility matrix and vice-versa.

ie $k = \frac{1}{f}$ or $k = \frac{1}{f}$

g. The stiffness co-efficient $K_{ij}$ is defined as the force developed at joint 'i' due to a unit displacement at joint 'j', wh[illegible] other joints are assumed to be fixed.

h. Displacements are considered as the unknown quantities

i. All the steps in the stiffness method can b[illegible] executed by a computer more conveniently. As the solution procedure is unique. It do[illegible] depend on the user for the choice of the primary structure. Hence, programming is simplified. The structure stiffness matrix for a stable linear structure is invariably a well-conditioned matrix, thereby facilitating accurate solutions of the response.

j. $i = (6m + r) - 6j$

External indeterminacy = 18.

Internal indeterminacy = 6.

# PART - B

2a)

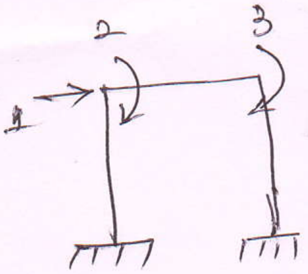



fig.1

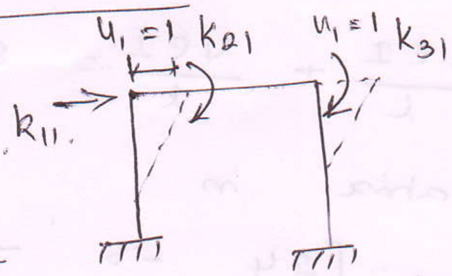



fig. 2

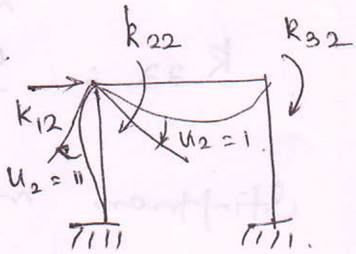



fig. 3

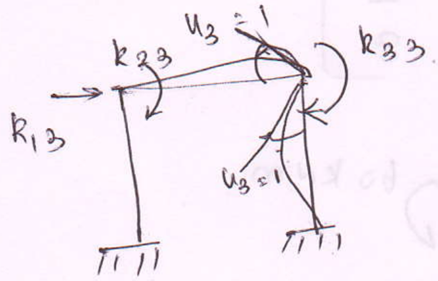



fig. 4

Fig.1 stiffness matrix to be developed for the co.ord. 1, 2 and 3.

To generate 1st column of the stiffness matrix, apply unit displacement @ coordinate 1 as shown in fig. 2.

$$k_{11} = \frac{12EI}{l^3} + \frac{12EI}{l^3} = 24.$$

$$k_{31} = \frac{-6EI}{l^2} = -6.$$

$$k_{21} = \frac{-6EI}{l^2} = -6 \; ;$$

To generate 2nd column, apply unit displacement at 2.

$$k_{12} = \frac{-6EI}{l^2} = -6 \; ; \quad k_{22} = \frac{4EI}{l} + \frac{4EI}{l} = 8$$

$$k_{32} = \frac{2EI}{l} = 2.$$

To generate 3rd column, apply unit displacement at 3

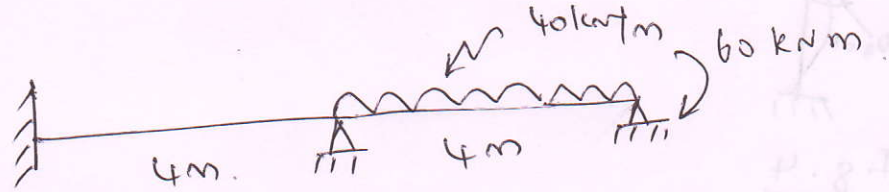
$$k_{13} = \frac{-6EI}{l^2} = -6; \quad k_{23} = \frac{2EI}{l} = 2;$$

$$k_{33} = \frac{4EI}{l} + \frac{4EI}{l} = 8.$$

Stiffness matrix is

$$[k] = \begin{bmatrix} 24 & -6 & -6 \\ -6 & 8 & 2 \\ -6 & 2 & 8 \end{bmatrix}$$

3a.



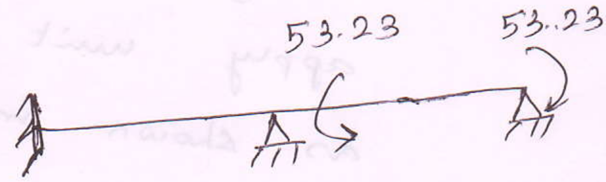

(1) Fixed end moments.

$$M_{FA} = M_{FBA} = 0$$

$$M_{FBC} = M_{FCB} = -\frac{40 \times 4^2}{12} = -53.23 \text{ kNm}$$

$$= +53.23 \text{ kNm}$$

$$\{P^0\} = \begin{Bmatrix} 0 \\ 0 \\ -53.23 \\ +53.23 \end{Bmatrix} \text{kNm}$$

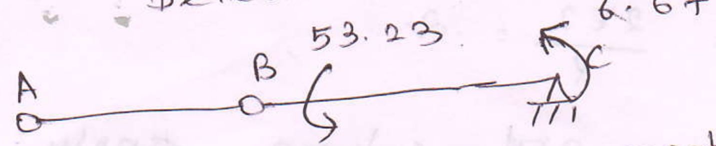



Fixed end moments

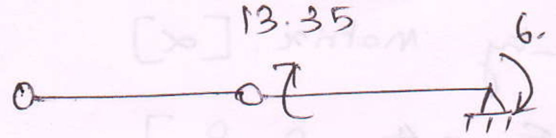
(2) Static indeterminacy = 2

Introduce hinges at A and B

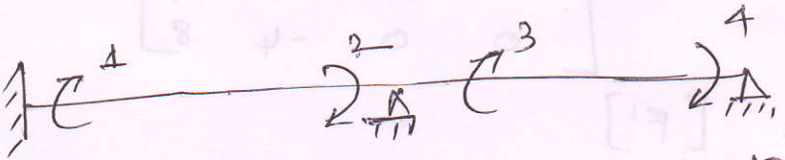

Determinate structure.



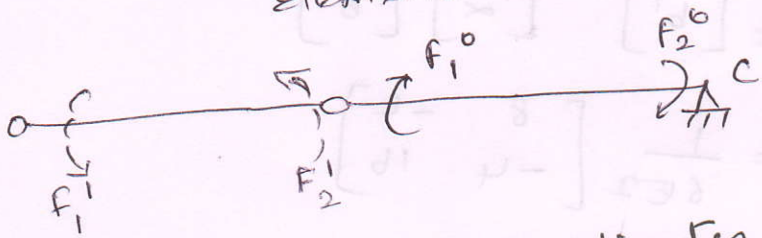

Unbalanced moment.




Balanced moment.




Element co-ordinates.

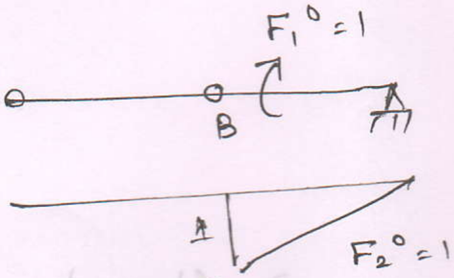


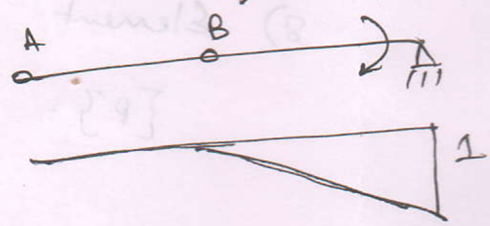

system coordinates

2) $[F^0] = \begin{bmatrix} 53.33 \\ 6.67 \end{bmatrix}$

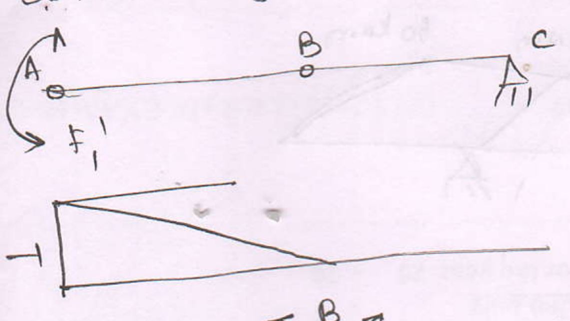
3) Apply unit forces (moment) $F_1^0$ & $F_2^0$ @ 3rd & 4th co-ordinates separately

$$[b^0] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

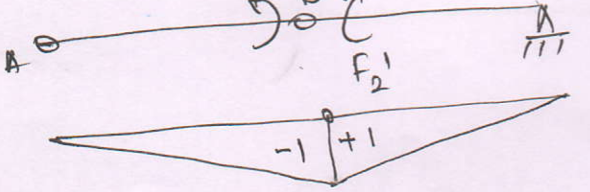



4) Apply unit redundant forces $F_1^1$ & $F_2^1$ to generate 1st & 2nd columns of $[b^1]$




$$[b^1] = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$



5) Element flexibility matrix. $[\alpha]$

$$[\alpha] = \frac{1}{6EI}\begin{bmatrix} 8 & -4 & 0 & 0 \\ -4 & 8 & 0 & 0 \\ 0 & 0 & 8 & 4 \\ 0 & 0 & -4 & 8 \end{bmatrix}$$

6) To generate $[F']$

$$[a_{11}] = [b']^T [\alpha] [b']$$

$$= \frac{1}{6EI}\begin{bmatrix} 8 & -4 \\ -4 & 16 \end{bmatrix}$$

7) $$\{F'\} = -[a_{11}]^{-1} [a_{10}] \{F^0\}.$$

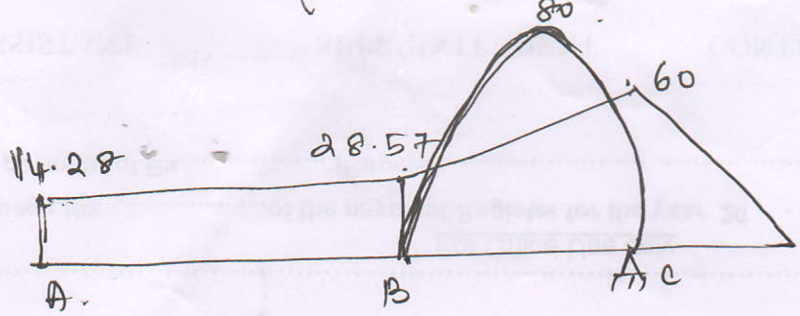
$$= -[a_{11}]^{-1} [b']^T [\alpha] [b^0] \{F^0\}$$

$$= \begin{Bmatrix} -14.28 \\ -28.57 \end{Bmatrix}$$

8) Element forces $\{P\}$

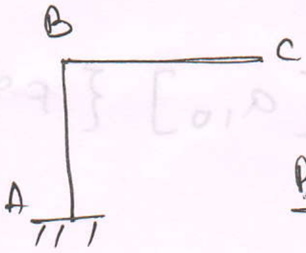
$$\{P\} = [b^0]\{F^0\} + [b']\{F'\} + \{P^0\}.$$

$$= \begin{Bmatrix} 14.28 \\ 28.57 \\ -28.57 \\ 60 \end{Bmatrix} \text{ kNm.}$$



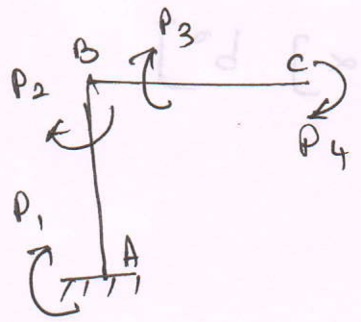

BMD.

4a) Static indeterminacy = 1

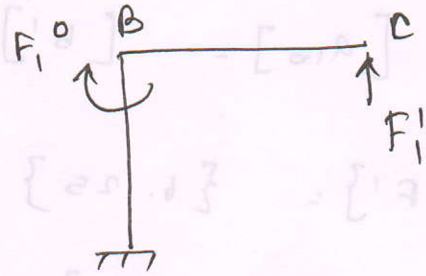



Primary structure

Consider ~~the~~ reaction at C as redundent force.

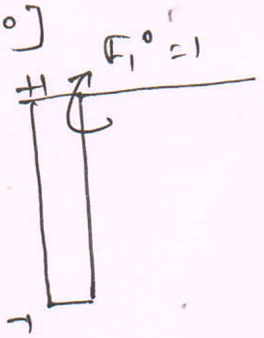



Element co.ord



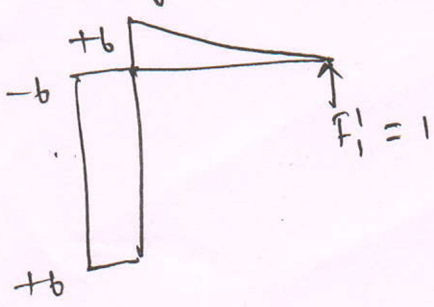

System co.ord

Apply unit external force $F_1^0$ at B to generate [$b^0$]




$$[b^0] = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Apply unit redundant force $F_1^1$ to generate [$b^1$]




$$[b^1] = \begin{bmatrix} 6 \\ -6 \\ 6 \\ 0 \end{bmatrix}$$

Element flexibility matrix [$\alpha$]

$$[\alpha] = \frac{6}{6EI} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

To generate $\{F^1\}$

$$\{F^1\} = -[a_{11}]^{-1} [a_{10}] \{F^0\}$$

Where

$$[a_{11}] = [b^1]^T [\alpha] [b^1]$$

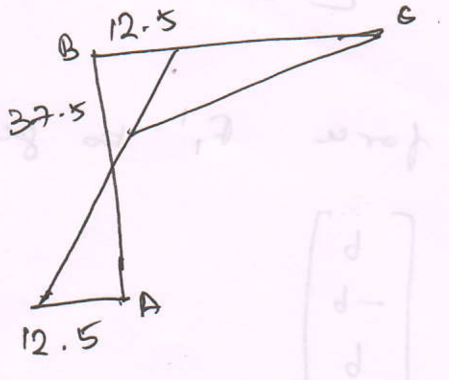
$$[a_{10}] = [b^1]^T [\alpha] b^0]$$

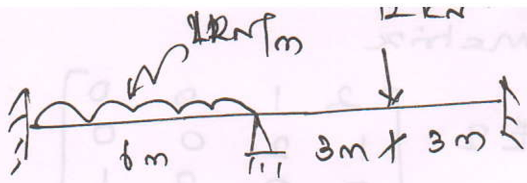
$$\{F^1\} = \{6.25\}$$

Element forces $\{P\}$

$$\{P\} = [b^0] \{F^0\} + [b^1] \{F^1\}$$

$$= \begin{Bmatrix} -12.5 \\ 12.5 \\ 37.5 \\ 0 \end{Bmatrix} \text{kNm.}$$



5a) 


Fixed end moments, $\{P^0\}$

$M_{FAB} = \dfrac{-2 \times 6^2}{12} = -6 \text{ kNm}$

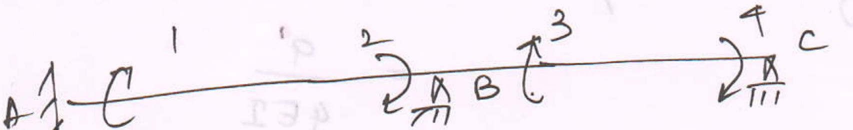
$M_{FBA} = +6 \text{ kNm}$

$M_{FBC} = \dfrac{-12 \times 6}{8} = -9 \text{ kNm}$

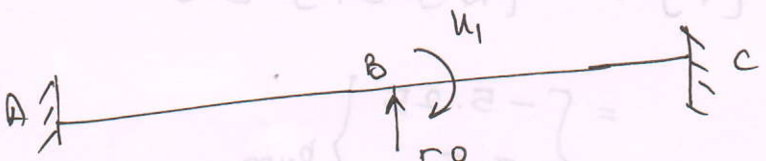
$M_{FCB} = +9 \text{ kNm}$

$$\{P^0\} = \begin{Bmatrix} -6 \\ 6 \\ -9 \\ 9 \end{Bmatrix}$$

Kinematic indeterminacy = 1

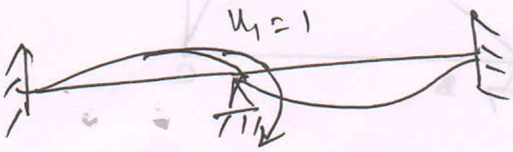



Element co.ord




System co.ord

Apply unit displacement to generate [β]




$$[\beta] = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Element stiffness matrix

$$[k_r] = EI \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

System stiffness matrix

$$[K] = [\beta]^T [k_r] [\beta]$$

$$= \frac{4EI}{3}$$

$$[K]^{-1} = \frac{3}{4EI}$$

To generate $\{\bar{F}\} = \{F\}^f - [\beta]^T \{P^0\}$

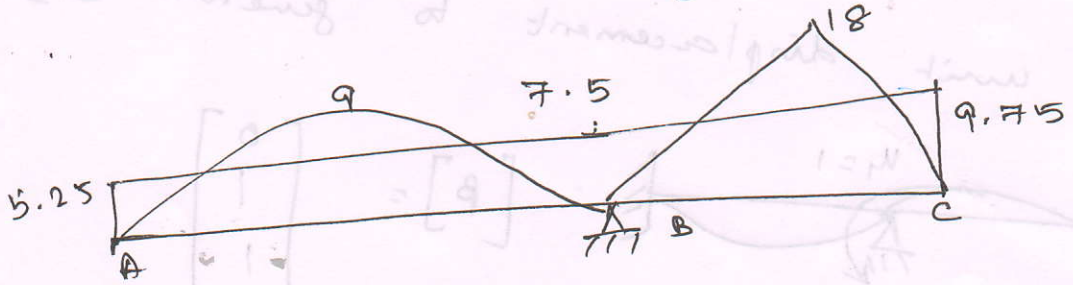
$$= \{3\}$$

System deformation, $\{u\} = [K]^{-1} \{\bar{F}\}$

$$= \frac{9}{4EI}$$

Element forces, $\{P\} = [k_r][\beta]\{u\} + \{P^0\}$

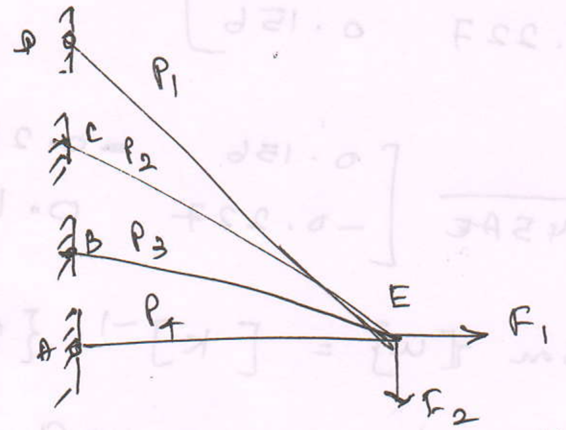
$$= \begin{Bmatrix} -5.25 \\ 7.5 \\ -7.5 \\ 9.75 \end{Bmatrix} \text{kNm}$$




BMD

6a) Degrees of freedom = 2.

~~Boot~~ ~~ian~~ ..




System co-ord & Element co.ord.

External load vector, $\{F\} = \begin{Bmatrix} 200 \\ 100 \end{Bmatrix}$

To generate force transformation matrix. $[E]$

Consider equilibrium of joint E

$F_1 = P_1 \cos 45^\circ + P_2 \cos 33.69^\circ + P_3 \cos 18.43^\circ + P_4$

$F_2 = P_1 \sin 45^\circ + P_2 \sin 33.69 + P_3 \sin 18.43^\circ$

$$[E] = [\beta]^T \begin{bmatrix} 0.707 & 0.707 \\ 0.832 & 0.555 \\ 0.949 & 0.316 \\ 1 & 0 \end{bmatrix}$$

To generate element stiffness matrix $[k]$

$$[k] = AE \begin{bmatrix} 1/6.364 & 0 & 0 & 0 \\ 0 & 1/5.408 & 0 & 0 \\ 0 & 0 & 1/4.743 & 0 \\ 0 & 0 & 0 & 1/4.5 \end{bmatrix}$$

System stiffness matrix $[K] = [\beta]^T [k] [\beta]$

$$= AE \begin{bmatrix} 0.618 & 0.227 \\ 0.227 & 0.156 \end{bmatrix}$$

$$[K]^{-1} = \frac{1}{0.045AE} \begin{bmatrix} 0.156 & -0.227 \\ -0.227 & 0.168 \end{bmatrix}$$

System deformation $\{u\} = [K]^{-1} \{F\}$.

$$= \frac{1}{AE} \begin{Bmatrix} 188.89 \\ -635.55 \end{Bmatrix}$$

Element forces $\{P\} = [k] [\beta] \{u\}$

$$= \begin{Bmatrix} -49.69 \\ -36.22 \\ -4.23 \\ 41.93 \end{Bmatrix} kN.$$

7a) Fixed end moments $[P^0]$

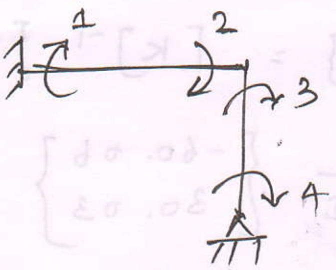
$M_{FAB} = \dfrac{-wl^2}{12} = -180\ kNm.$

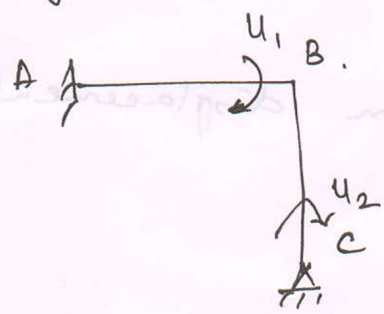
$M_{FBA} = +180\ kNm.$

$M_{FBC} = M_{FCB} = 0$

$$\{P^0\} = \begin{Bmatrix} -180 \\ +180 \\ 0 \\ 0 \end{Bmatrix}$$

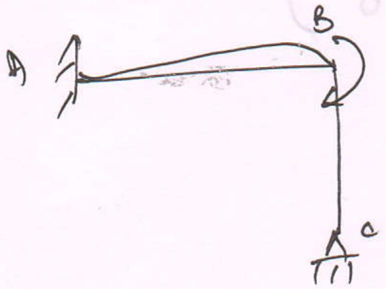
Kinematic indeterminacy = 2.

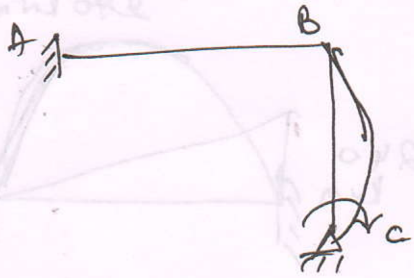



Element co.ord System co.ord

Apply unit displ. @ 3rd & 4th co.ord. to generate 1st & 2nd columns. of [β].




$$[\beta] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Element stiffness matrix, $[k] = EI \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 4/3 & 2/3 \\ 0 & 0 & 2/3 & 4/3 \end{bmatrix}$

System stiffness matrix, $[K] = [\beta]^T [k] [\beta]$

$$[K] = EI \begin{bmatrix} 10/3 & 2/3 \\ 2/3 & 4/3 \end{bmatrix}$$

$$[K]^{-1} = \frac{1}{4EI} \begin{bmatrix} 4/3 & -2/3 \\ -2/3 & 10/3 \end{bmatrix}$$

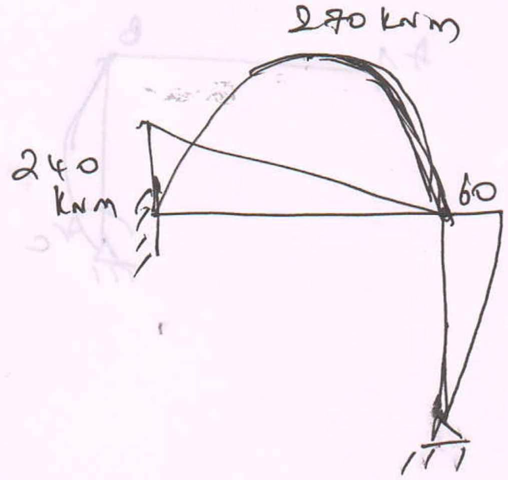
$$\{\bar{F}\} = \{F\}^{f} - [\beta]^{T}\{P^{0}\} = \begin{Bmatrix} -180 \\ 0 \end{Bmatrix} kN$$

System displacement, $\{u\} = [K]^{-1}\{\bar{F}\}$

$$= \frac{1}{EI}\begin{Bmatrix} -60.06 \\ 30.03 \end{Bmatrix}$$

Element forces, $\{P\} = [k][\beta]\{u\} + \{P^{0}\}$

$$= \begin{Bmatrix} -240 \\ 60 \\ -60 \\ 0 \end{Bmatrix}$$




BMD

8. a) Techniques used to reduce memory requirement for storing the stiffness matrix

(1) Use of symmetry & banded nature

(2) Partitioning of matrix

(3) Skyline storage.

Explanation on the above types.