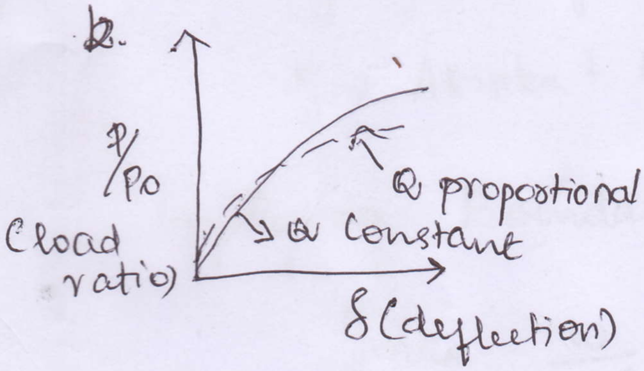
MSEPC1020 – Set 1 Answers

Elastic Stability and Behaviour of Metal Stru

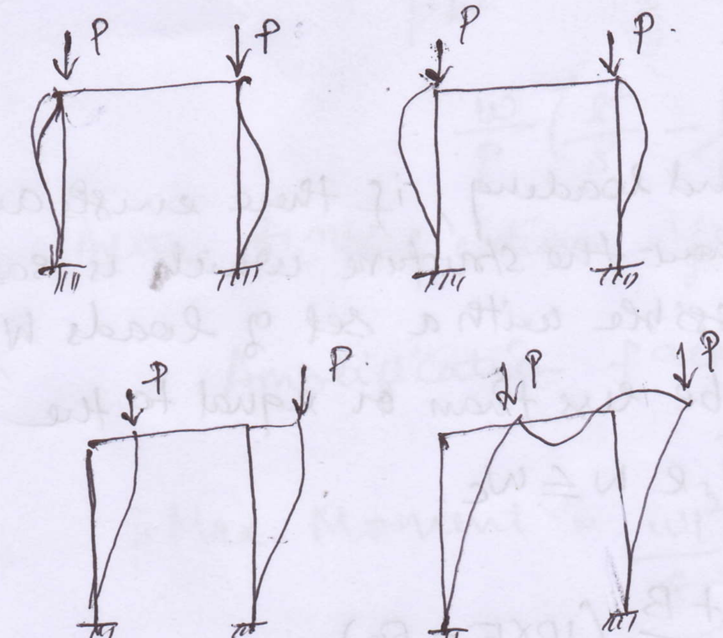
PART A (10×2 = 20)

a. Lateral Buckling & bending

b.



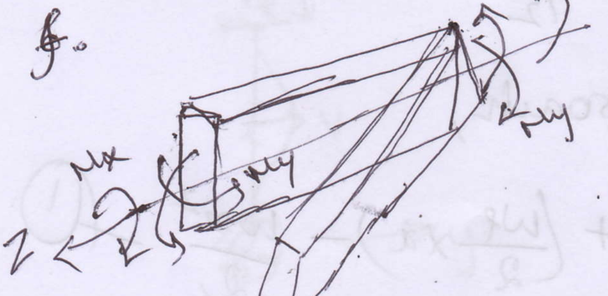
c.



d. $T = GJ\dfrac{d\beta}{dz}$

e. when torsional flexural buckling is significantly sma
than euler load

f.



9. 1. Transverse deflections of the plates are small compared thickness of the plate

2. Material of the plate is homogeneous, isotropic and obey's Hooke's law

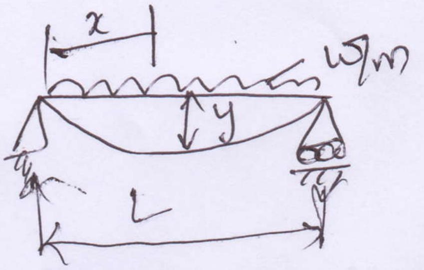
8. ductile and fatigue failure

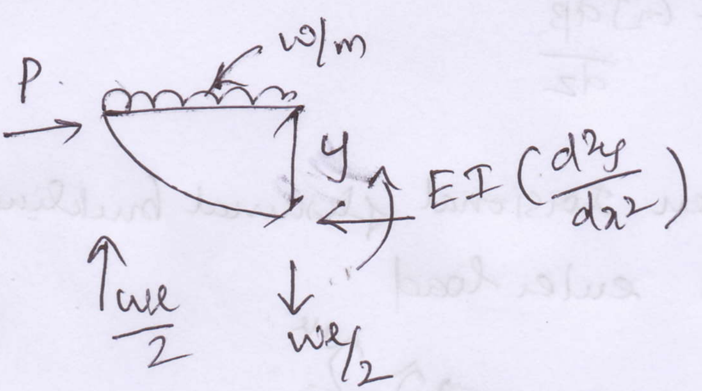
i. $S = \dfrac{M_p}{M_y} = \dfrac{\sigma_y Z_p}{\sigma_y Z} = \dfrac{Z_p}{Z}$

j. For a given structure and loading, if there exists any distribution of BM throughout the structure which is safe and statically admissible with a set of loads W, the value of W must be less than or equal to the collapse load Wc. i.e $W \leq W_c$

Part B (10×5 = 50)

2a)





Using equilibrium approach,

$$-EI\frac{d^2y}{dx^2} = Py + \left(\frac{wl}{2}\times x\right) - \frac{wx^2}{2} \quad (1)$$

$\div EI$, $\quad y'' + \left(\frac{P}{EI}\right) y = \frac{w}{2EI}(x^2 - Lx)$

$$\frac{P}{EI} = k^2.$$

Solving ~~Axillary~~ Axillary equation and particular Intigral

$$y = A\sin kx + B\cos kx + \frac{w}{P}\left(\frac{x^2}{2} - \frac{1}{k^2} - \frac{xl}{2}\right)$$

Applying Boundary conditions,

$$y_{max} = \frac{w}{pk^2} \tan\frac{kl}{2}\left(\sin\frac{kl}{2}\right) + \frac{w}{pk^2}\left(\cos\frac{kl}{2}\right) +$$

$$\frac{w}{P}\left(\frac{l^2}{8} - \frac{1}{k^2} - \frac{l^2}{4}\right]$$

After simplification $\quad y_{max} = y_0\left(\frac{1}{1-\left(\frac{P}{P_{cr}}\right)}\right]$ (4)

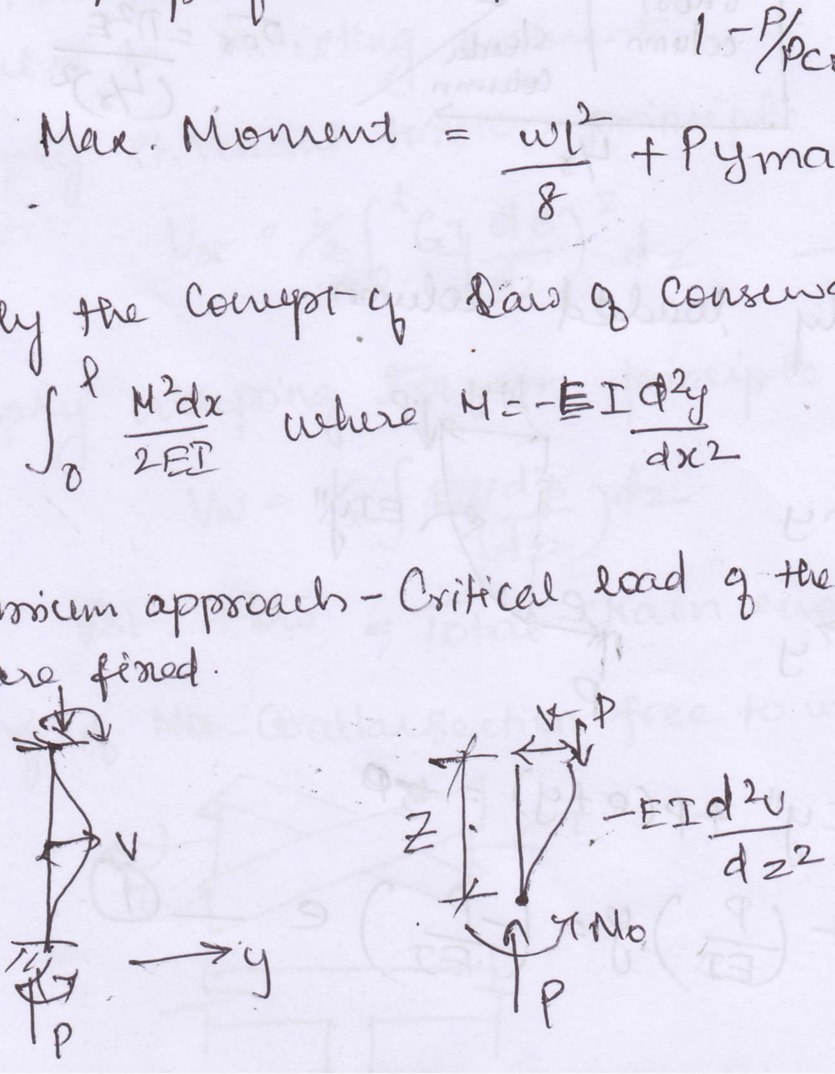
Amplification factor $= \dfrac{1}{1 - P/P_{cr}}$ (2)

Max. Moment $= \dfrac{wl^2}{8} + P y_{max}$ (2)

b) (1) Apply the Concept of Law of Conservation of energy (2

(2) $U = \int_0^l \frac{M^2 dx}{2EI}$ where $M = -EI\frac{d^2y}{dx^2}$ (1

3a. Equilibrium approach - Critical load of the column when both ends are fixed.



Internally resisting moment

$$M = -EI\frac{d^2v}{dz^2} \quad \text{—(1)}$$

Externally resisting moment $= -M_0 + Pv$ —(2)

$$-M_0 + Pv = -EI\frac{d^2v}{dz^2} \qquad (2)$$

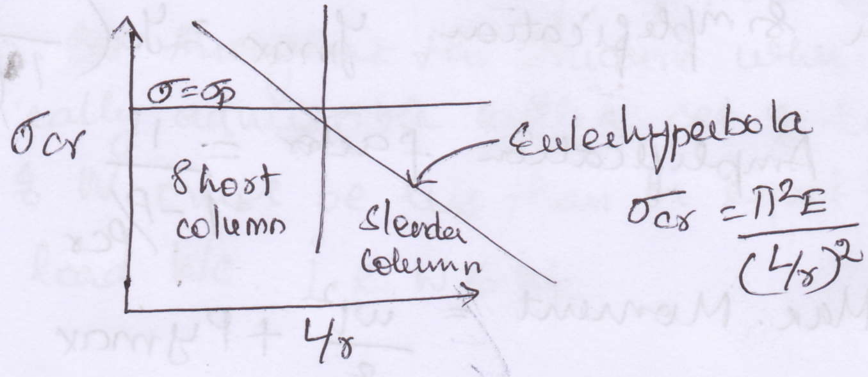
Solving these,

$$v = A\sin kz + B\cos kz + \frac{M_0}{P}$$

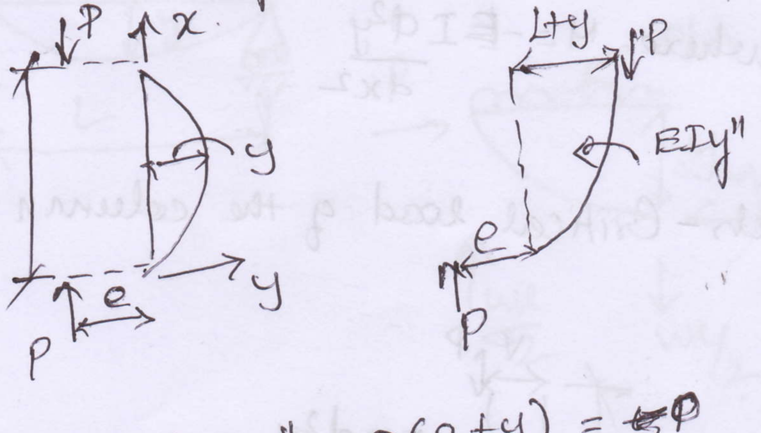
Apply Boundary conditions, (3)

$$P_{cr} = \frac{\pi^2 EI}{Le^2} \quad \text{where } Le = L/2$$

b) concept of Inelastic analysis of columns (3)



a) Eccentrically loaded column.



$$EIy'' + P(e+y) = 0$$

$$y'' + \left(\frac{P}{EI}\right)y = \left(\frac{-P}{EI}\right)e \quad \text{—(1)}$$

Solving the above eqn ①

$$y = A\cos kx + B\sin kx + e$$

$$\text{Max deflection} = e\left[\sec\sqrt{\frac{P}{P_E}}\left(\frac{\pi}{2}\right) - 1\right] \quad \text{— (4)}$$

$$\text{Max Bending Moment} = P\left(e \cdot \sec\frac{kl}{2}\right) \quad \text{— (4)}$$

b.

Tangent Modulus	Reduced Modulus
(1) No strain reversal	Strain Reversal
(2) Axial load is assumed to increase during transition from the straight to bent	Axial load is constant

(2)

5a. Expression for total strain energy stored in a member subjected to twisting moment

(i) Apply St-Venant torsion principle.

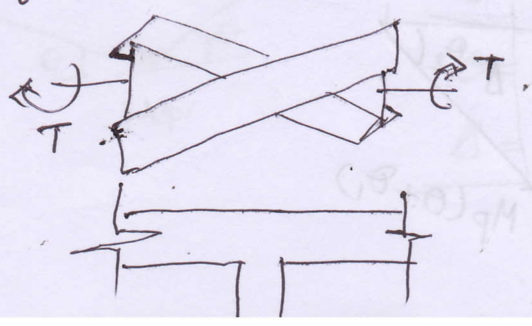
$$U_{st} = \frac{1}{2}\int_0^l GJ\left(\frac{d\beta}{dz}\right)^2 dz \quad \text{— (4)}$$

(ii) Apply Warping torsion principle.

$$U_w = \frac{1}{2}\int EI\left(\frac{d^2\beta}{dz^2}\right) dz \quad \text{— (4)}$$

$$U_{st} + U_w = \text{Total strain energy}$$

b) Twisting of Non-Circular section free to wrap. (2)



6a) Critical stress developed due to lateral buckling of simply supported beam in pure bending.

$$u = v = \frac{d^2u}{dz^3} = \frac{d^2v}{dz^2} = 0 \quad at\ z = 0, l \qquad -(2)$$

$$\beta = \frac{d^2\beta}{dz^2} = 0 \quad at\ z = 0, l$$

(2)

$$GJ\frac{d\beta}{dz} - M_z\frac{du}{dz} = 0$$

$$k^2 = M^2/GJEI_r \implies$$

$$\frac{d^2\beta}{dz^2} + k^2\beta = 0 \implies \text{solution this equation}$$

(2)

$$\beta = A\sin kz + B\cos kz$$

Solving this

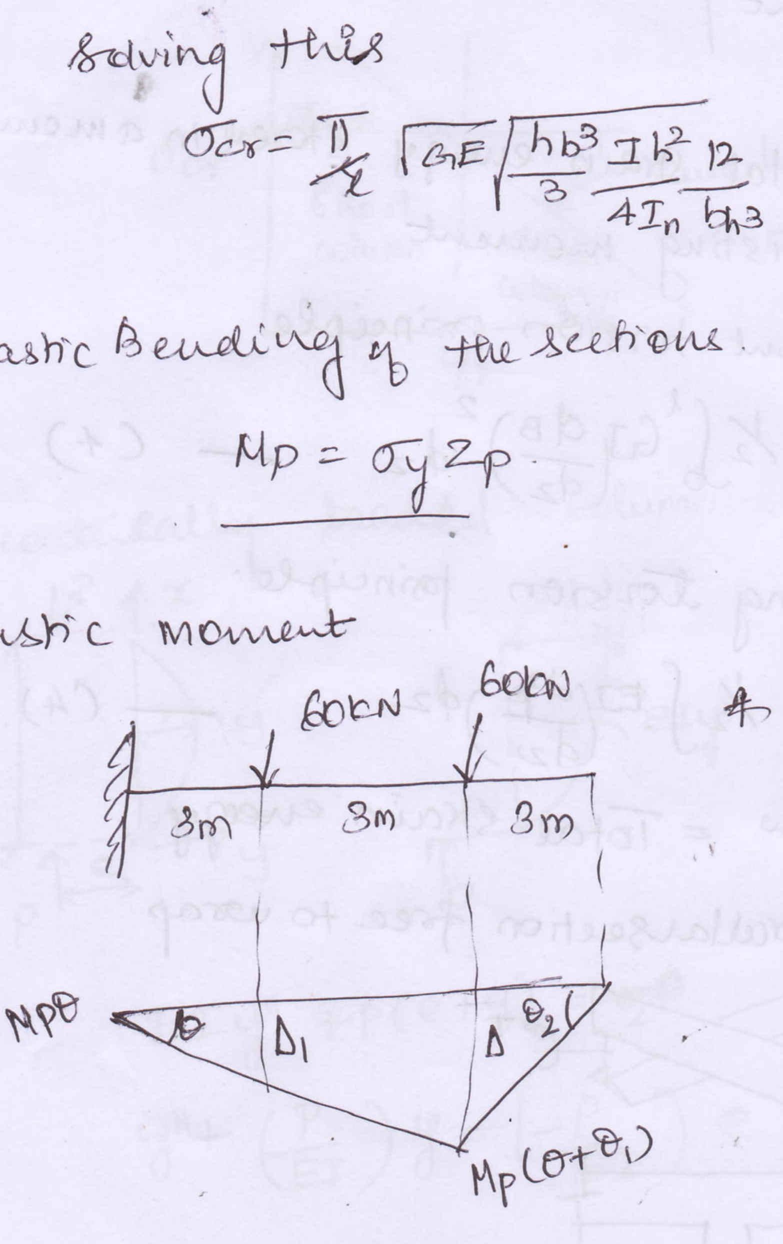
$$\sigma_{cr} = \frac{\pi}{l}\sqrt{GE}\sqrt{\frac{hb^3}{3}\frac{Ib^2}{4I_n}\frac{12}{bh^3}} = \frac{\pi\sqrt{GE}}{I/b}\sqrt{\frac{I_p}{I_n}} \quad (2)$$

b) Plastic Bending of the sections

(2)

$$\underline{M_p = \sigma_y Z_p}$$

7a. Plastic moment



$\tan\theta = \Delta/6$ | $\tan\theta_1 = \Delta/3$ (2)

$\theta = \Delta/6$ | $\theta_1 = \Delta/3$

$\Delta = 6\theta$ | $\Delta = 3\theta_1$

$3\theta_1 = 6\theta$

$\theta_1 = 2\theta$

$\Delta_1 = 3\theta$ (2)

Internal workdon $= 4M_p\theta$ —(1) (2)

External workdone $= 540\theta$ —(2)

(1) = (2) (2)

$\boxed{M_p = 135\,kNm}$

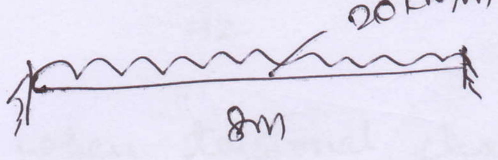
b) concept of plastic analysis

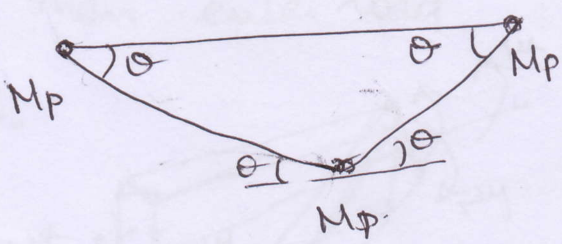
(i) $\frac{M_y}{I} = \frac{\sigma_y}{y} \Rightarrow M_y = \sigma_y \times I_y \Rightarrow M_y = \sigma_y \times Z$ (2)

$M_p = \sigma_y . Z_p$

(2) If the section is homogeneous and symmetried, there not be any shift in NA during the plastic state (2

8a,





$\tan\theta = \Delta/4$

θ is very small

$\theta = \Delta/4$

$\Delta = 4\theta$ (2)

Internal work done $= 4 M_p \theta$ —— (1) (2)

External work done $= 320\theta$ —— (2) (2)

$$4 M_p \theta = 320\theta$$

$$\boxed{M_p = 80 kNm}$$ (2)

b) Assumptions made in plastic analysis. (2)

(1) Plane transverse sections remain plane and normal to longitudinal axis after bending.

(2) Material - homogeneous, isotropic - elastic & plastic sta