QPC: RM19MSC159 AR 19 Reg. No.

GIET UNIVERSITY, GUNUPUR – 765022

M. Sc. (Fourth Semester) Examinations, May – 2021

MTPE 405 – ORDINARY DIFFERENTIAL QUATIONS – II (MATHEMATICS)

Time: 2 hrs Maximum: 50 Marks

(The figures in the right hand margin indicate marks.)

PART – A **(2 x 10 = 20)**

Q.2. Answer **ALL** the questions

a. Define equicontinuous.

b. State Ascoli's lemma.

c. Define regular linear boundary value problem.

d. Is a differential equation $x''+|x|=0,\ 0\le t\le \pi$ with boundary conditions $x(0)=x(\pi)=0$ is linear.

e. State Sturm's separation theorem.

f. State Wintner lemma.

g. Define repeller.

h. Define asymptotically stable.

i. Show that the equation $x''+\dfrac{x}{1+t}=0,\ t\ge 0$ is oscillatory.

j. Discuss the nature of the critical points for $x_1'=x_1+2x_2,\ x_2'=-2x_1+5x_2$

PART – B **(6 x 5 = 30 Marks)**

<u>Answer ***ANY FIVE*** questions</u> Marks

2. State and Prove Alekseev's formula (6)

3. Let $I=[t_o, t_o+h]$, $v,w\in C^1[I,R]$ be lower and upper solutions of $x'=f(t,x),\ x(t_o)=x_0$ such that $v(t)\le w(t)$ on I and $f\in C[\Omega,R]$. Then prove that there exists a solution $x(t)$ of $x'=f(t,x),\ x(t_o)=x_0$ such that $v(t)\le x(t)\le w(t)$ on I. (6)

4. State and Prove Picard's theorem (6)

5. Use the method of separation of variables to solve the boundary value problem (6)

$$\frac{\partial u}{\partial t}(x,t)=h^2\frac{\partial^2 u}{\partial x^2}(x,t),\ 0<t<1,\ t>0.$$

$$u(x,0)=u_0x,\ 0<x<1,\ u(0,t)=u(1,t)=0,\ t>0.$$

6. Prove that the zeros of a solution of $x''+a(t)x'+b(t)x=0,\ t\ge 0$ are isolated. (6)

7. State and Prove Sturm's comparison theorem. (6)

8. Let all solutions of the equation $x' = A(t)x$ be bounded. Let (6)

(i) $\int_0^\infty \|B(s)\| ds < \infty$

(ii) $\lim_{t \to \infty} \int_0^1 Tr\, A(s) ds > -\infty$.

Then prove that all the solutions of $y' = [A(t) + B(t)]y$ are bounded.

9. State and prove Liapunov's first theorem on stability. (6)

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