QP Code: RM19MSC137 Reg. No | | | | | | | | | | AR 19

GIET UNIVERSITY, GUNUPUR – 765022

M. Sc (Fourth Semester) Examinations, May' 2021

MTPC 402 – FUNCTIONAL ANALYSIS-II

(Mathematics)

Time: 2 hrs Maximum: 50 Marks

(The figures in the right hand margin indicate marks.)

PART – A **(2 × 10 = 20 Marks)**

Q.1. Answer **ALL** questions

a. Define weak convergent in a normed space X and provide an example for it.
b. State Bolzono - Weierstrass theorem.
c. Define an inner product space.
d. Define the terms orthogonal vectors and orthonormal set in an inner product space.
e. Define an optimal error.
f. Write the complemented subspace property of the Hilbert space H.
g. Define bonded operator in a Hilbert space H.
h. Why an unitary operator is called as Hilbert space isomorphism.
i. State Polarization identity in an inner product space.
j. State Schwarz inequality in an inner product space.

PART-B **(6 × 5 = 30 Marks)**

Answer **ANY FIVE** questions Marks

2. Let X be a normed space. Then prove that X is reflexive if and only if every bounded sequence in X has a weak convergent subsequence. (6)

3. State and prove the Bessel's inequality. (6)

4. State and prove the theorem on Gram-Schmidt orthonormalization. (6)

5. State and prove Riesz- Fischer theorem. (6)

6. Let $\{ x_1,\dots, x_n \}$ be an orthogonal set in an inner product space X. Then prove that $\| x_1+\dots+x_n \|^2 = \| x_1\|^2 +\dots + \| x_n \|^2$. (6)

7. Prove that a subset of a Hilbert space is weak bounded if and only if it is bounded. (6)

8. Let H be a Hilbert space and $A \in BL(H)$. Then prove that there is a unique $B \in BL(H)$ such that for all $x, y \in H$, $< A(x), y > = < x, B(y) >$. (6)

9. Let H be a Hilbert space with $A \in BL(H)$. Let A be self-adjoint. Then prove that $\| A \|| = \text{Sup} \{ |< A(x), x >| : x \in H , \| x \| =1 \}$. Also prove that A = 0 if and only if $< A(x), x > =0$ for all $x \in H$. (6)

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