QP Code: RS20MSC149 Reg. No | | | | | | | | | | AR 20

GIET UNIVERSITY, GUNUPUR – 765022

M. Sc. (Second Semester) Examinations, September – 2021

20MTPC203 – Partial Differential Equations

(Mathematics)

Time: 2 hrs Maximum: 50 Marks

(The figures in the right hand margin indicate marks.)

PART – A

Q.1. Answer ***ALL*** questions **(2 x 10 = 20 Marks)**

a. Find the tangent vector at $(0,1,\pi/2)$ to the helix described by the equation $x = \cos t,\ y = \sin t, z = t, t \in I$ in R'.

b. Form the PDE by eliminating the arbitrary function from $f(x+y+z, x^2+y^2+z^2) = 0$.

c. Find the general solution of the PDE $xz\,p + yz\,q = xy$.

d. Show that the PDEs $xp - yq = x$ and $x^2p + q = xz$ are compatible.

e. Define Dirac delta function and write any two properties.

f. What is the form of diffusion equation in cylindrical coordinates?

g. Define elliptic differential equation with an example.

h. Classify and reduce the relation $y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y$ to a canonical form.

i. How many boundary conditions are there in heat conduction equation? Discuss about each type of boundary condition.

j. Define Cauchy problem for first order equations.

PART – B (6 x 5 = 30 Marks)

Answer ***ANY FIVE*** the questions Marks

2. Find the general integral of the following linear PDEs (6)

$(x^2+y^2)p + 2xy\,q = (x+y)z$

3. Solve the one-dimensional diffusion equation (6)

$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}, -\infty < x < \infty, t > 0$ with **IC**: $T(x,0) = f(x), -\infty < x < \infty, t = 0.$

4. State and solve the Neumann problem for a rectangle. (6)

5. State and solve interior Dirichlet problem for a circle. (6)

6. The ends A and B of a rod, 10 cm in length, are kept at temperature $0^\circ C$ and $100^\circ C$ until the steady state condition prevails. Suddenly the temperature at the end A is increased to $20^\circ C$, and the end B is decreased to $60^\circ C$. Find the temperature distribution in the rod at time t. (6)

7. Find the temperature in a sphere of radius a, when its surface is kept at zero temperature and its initial temperature is $f(r,\theta)$ (6)

8. Solve the following PDE using Laplace transform. (6)

$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, x > 0, t > 0$ with $u(0,t) = u_0$(constant)

when $t > 0$ and $u(x,0) = 0$ when $x > 0$.

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