QP Code: RS20MSC135 Reg. No | | | | | | | | | | AR 20

GIET UNIVERSITY, GUNUPUR – 765022

M. Sc. (Second Semester) Examinations, September – 2021

20MTPC201 – Algebra II

(Mathematics)

Time: 2 hrs Maximum: 50 Marks

The figures in the right-hand margin indicate marks
All the symbols have their usual meaning

PART- A

Q.1: Answer ALL questions ($2 \times 10 = 20$ Marks)

a. If $\dim_F V = m$, then what is the value of $\dim_F Hom(V, V)$ and $\dim_F Hom(V, F)$?

b. If $v \in V$, then define the norm of v, and prove that $\|\alpha v\| = |\alpha|\|v\|$.

c. If G be the group of automorphisms of K, then define the fixed field of G, and what is its relation with K?

d. Define the solvability of a group G.

e. Define the characteristic root and characteristic vector of $T \in A(V)$.

f. If $T, S \in A(V)$ and S is regular, then show that T and STS^{-1} have the same minimal polynomial.

g. Define the similarity of linear transformations $S, T \in A(V)$.

h. If $T \in A(V)$ is nilpotent, then define the index of nilpotence.

i. If $T \in A(V)$ and if $S \in A(V)$ is regular, then show that $r(T) = r(STS^{-1})$.

j. If $(vT, vT) = (v, v)$ for all $v \in V$, then what is the nature of T?

PART- B

Answer ANY FIVE questions **($6 \times 5 = 50$ Marks)**

2. If V and W are of dimension m and n, respectively, over F, then show that $Hom(V, W)$ is of dimension mn over F.

3. If $u, v \in V$, then show that $|(u, v)| \le \|u\|\|v\|$.

4. If K is a finite extension of F, then prove that $G(K, F)$ is a finite group and its order, $o(G(K, F))$ satisfies $o(G(K, F)) \le [K : F]$.

5. Show that S_n is not solvable for $n \ge 5$ followed by the proof of “Let $G = S_n$, where $n \ge 5$; then $G^{(k)}$ for $k = 1, 2, \cdots$, contains every 3-cycle of S_n.

6. If A is an algebra, with unit element, over F, then show that A is isomorphic to asubgroup of $A(V)$ for some vector space V over F.

7. (a) Define the trace of A, and prove that for $A, B \in F_n$ and $\lambda \in F$,

   i. $tr(\lambda A) = \lambda\ tr\ A$

   ii. $tr(A + B) = tr\ A + tr\ B$

   iii. $tr(AB) = tr(BA)$.

8. If M, of dimension m, is cyclic with respect to T, then the dimension of MT^k is $m - k$ for all $k \le m$.

———— End of Paper ————