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GIET UNIVERSITY, GUNUPUR – 765022
M. Sc. (First Semester) Examinations, May – 2021
20MTPC104 – INTEGRAL TRANSFORMATIONS
(Mathematics)

Time: 2 hours

Maximum: 50 Marks

(The figures in the right hand margin indicate marks.)

PART – A**(2 x 10 = 20 Marks)**Q.1. Answer **ALL** the questions

- Find the Laplace Trans form of $\cos^2 2t$.
- Find the inverse Laplace transform of $\log\left(\frac{s+1}{s-1}\right)$.
- Find the Laplace Transform of the Bessel Function $J_0(x)$.
- Express $f(x) = \frac{x}{2}$ as a Fourier series in the interval $-\pi < x < \pi$.
- State and Prove the Shifting Property of Fourier Transforms.
- Using Parseval's Identity for Fourier Cosine Transforms, prove that $\int_0^\infty \frac{dt}{(a^2 + t^2)(b^2 + t^2)} = \frac{\pi}{2ab(a+b)}$
- Obtain the zero-order Hankel transform of $\frac{\exp(-ar)}{r}$.
- State and Prove the Parseval's Relation of Hankel Transforms.
- Find the Z Transform of e^{an} .
- State and Prove the Initial Value Theorem of Z Transforms.

PART – B**(6 x 5 = 30 Marks)**Answer **ANY FIVE** questions

Marks

- Using Laplace Transforms, solve the differential equation $y''' + 2y'' - y' - 2y = 0$ given $y(0) = y'(0) = 0$ and $y''(0) = 6$. (6)
- If $f(t)$ is a periodic function with period T , i.e., $f(t+T) = f(t)$, then show that (6)

$$L[f(t)] = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt.$$
- State and Prove Fourier Integral Theorem. (6)
- Find the Fourier Cosine Transform of $f(x) = \begin{cases} x, & \text{for } 0 < x < 1 \\ 2 - x, & \text{for } 1 < x < 2 \\ 0, & \text{for } x > 2 \end{cases}$ (6)
- Derive the Hankel Transform of Derivatives. (6)
- State and Prove the Convolution Theorem of Z Transforms. (6)
- Using Z Transforms, solve the difference equation $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$ with $u_0 = 0, u_1 = 1$. (6)

--- End of Paper ---