QP Code: RM20MSC099 Reg. No | | | | | | | | | | AR 20

GIET UNIVERSITY, GUNUPUR – 765022

M. Sc (First Semester) Examinations, May – 2021

20MTPC103 – ORDINARY DIFFERENTIAL EQUATION

(MATHEMATICS)

Time: 2 hrs Maximum: 50 Marks

(The figures in the right hand margin indicate marks.)

PART – A **(2 x 10 = 20 Marks)**

Q.1. Answer ***ALL*** questions

a. Find the differential equation by eliminating the arbitrary constant from the equation $a^3x + a^2t + 5 = 0$, a is a constant.

b. Solve { y(1 + 1 / x) + cos y}dx + (x + log x - x sin y) dy = 0 .

c. Find the Wronskian of the functions e^t, cos t and sin t.

d. Find a differential system for which the vector y(t) = $\begin{pmatrix} t^2 + 2t + 5 \\ sin^2 t \end{pmatrix}$, t ∈ I is a solution.

e. Find e^{At} , where A = $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

f. Determine the fundamental matrix for the system x' = Ax , where A = $\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$

g. Define oscillatory and non-oscillatory solutions with examples.

h. Prove that $x'' + (k / t^2 - e^{-t}) x = 0$; k > 1/ 4 is oscillatory.

i. Define attractor and repeller.

j. Determine the nature of the characteristic roots for the equation $x''' + 3x'' + 2x' + x = 0$

PART – B **(6 x 5 = 30 Marks)**

Answer ***ANY FIVE*** questions Marks

2. Show that e^{2x} and e^{3x} are linearly independent solutions of $\boldsymbol{y'' - 5y' + 6y = 0}$. Find the solution y(x) with the property that y(0) = 0 and $y'(0) = 1$. (6)

3. State Abel's formula and derive the expression for w(t). (6)

4. Suppose A(t) be an n × n matrix which is continuous on I and a matrix Φ satisfies $X' = A(t)X$, t ∈ I . Then show that (det Φ) satisfies the first order equation $(\det \Phi)' = (\mathrm{tr}\, A)(\det \Phi)$ (6)

5. State and prove Sturm's comparison theorem. (6)

6. Let a(t) be continuous and positive on (0, ∞) with $\int_1^\infty a(t)dt = \infty$. Assume that x(t) is any solution of $x'' + a(t)x = 0,\ t \geq 0$. Then show that it has infinite number of zeros. (6)

7. Define a stable matrix and prove that if the matrix A in $x' = Ax;\ 0 \le t < \infty$ is stable, then for any solution x(t) of equation $x' = Ax;\ 0 \le t < \infty$, $\lim_{t\to\infty} \| x(t) \| = 0$ (6)

8. Consider the equation $x'' + a(t)x = 0,\ 0 \le t < \infty$. Let $\int_1^\infty t|a(t)|dt < \infty$. Then show that limit $x'(t)$ exists and the general solution of equation $x'' + a(t)x = 0$, $0 \le t < \infty$ is asymptotic to $a_0 + a_1 t$, where a_0 and a_1 are constants. (6)

--- End of Paper ---