QP Code: RM20MSC087 Reg. No | | | | | | | | | | | AR 20

GIET UNIVERSITY, GUNUPUR – 765022

M. Sc(First Semester) Examinations, May – 2021

20MTPC101– ALGEBRA – I

(Mathematics)

Time: 2 hrs Maximum: 50 Marks

(The figures in the right hand margin indicate marks.)

PART – A **(2 x 10 = 20 Marks)**

Q.1. Answer ***ALL*** questions

a. Obtain the orbits and cycles of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$

b. State Cayley's theorem

c. Define conjugacy relation

d. Define a maximal ideal

e. Define an Euclidean Ring

f. Define primitive polynomial

g. Define a vector space

h. Define Linear span

i. State Remainder theorem

j. Define:

(i) Extension of a field and degree of such an extension

(ii) finite extension

PART – B **(6 x 5 = 30 Marks)**

<u>Answer ***ANY FIVE*** the questions</u> Marks

2. If G is a group, then prove that $A(G)$, the set of automorphisms of G is also a group. (6)
3. Show that conjugacy is an equivalence relation (6)
4. State and Prove Sylow's theorem (6)
5. State and Prove factorization theorem (6)
6. Given two polynomials $f(x)$ and $g(x) \neq 0$ in $F[x]$ then prove that there exist two polynomials $t(x)$ and $r(x)$ in $F[x]$ such that $f(x) = t(x)g(x) + r(x)$ where $r(x) = 0$ (or) $\deg r(x) < \deg g(x)$ (6)
7. If A and B are subspaces of a finite dimensional vector space V prove that $A+B$ is also finite dimensional and $\dim(A+B) = \dim(A) + \dim(B) - \dim(A \cap B)$ (6)
8. If K is a finite extension of F and L is a finite extension of K, then prove that L is a finite extension of F and $[L:F] = [L:K][K:F]$ (6)

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