QP Code: RD19MSC077 Reg. No | | | | | | | | | | AR 19

GIET UNIVERSITY, GUNUPUR – 765022

M. Sc (Third Semester) Examinations, December' 2020

CE 313 / MTPE 304 – ORDINARY DIFFERENTIAL EQUATIONS

(Mathematics)

Time: 2 hrs Maximum: 50 Marks

(The figures in the right hand margin indicate marks.)

Part – A

Q.1 Answer all questions from the following **[2 x 10 = 20]**

a Determine the order of the differential equation
$$\left(\frac{d^2y}{dx^2}\right)^3+7\left(\frac{d^2y}{dx^2}\right)^2 \Big/ \left(\frac{d^2y}{dx^2}+\frac{d^3y}{dx^3}\right)=y$$

b Define exact differential equation.

c Find the Wronskian $w(t)$ of the functions $e^t, \cos t, \sin t$.

d Find the general solution of $x''+2x'+3x=t^4+3,\ \ x(0)=0;\ x'(0)=1.$

e Find a fundamental matrix for the system $x'=Ax$,

where $A=\begin{bmatrix}\alpha_1 & 0 & 0\\ 0 & \alpha_2 & 0\\ 0 & 0 & \alpha_3\end{bmatrix}$, $\alpha_1,\ \alpha_2\ and\ \alpha_3$ are scalars.

f Let $f(x)=x^{1/2}$ be defined on the rectangle $R=\{(t,x):|t|\le 2,\ |x|\le 2\}$. Then prove that f does not satisfy Lipschitz condition in R.

g Write the characteristic equation of the delay equation $x'(t)=ax(t)+bx(t-r),\ \ 0\le t_o\le t<\infty.$

h Write initial value problem for a linear delay differential equation with constant coefficients having a constant delay $r>0$.

i. Solve the IVP, for $\pi/2\le t<\pi,\ \ x'+(\cot x)=2\text{cosec}t,\ \ x(\pi/2)=1.$

j. Find e^{At} when $A=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$

Part - B

Answer **ANY FIVE** Questions: **[6 x 5 = 30]**

2. Prove that the functions $1, t, t^2,\ldots t^n,\ -\infty<t<\infty$ are linearly independent. (6)

3. Prove that there are three linearly independent solutions of the third order equation $x'''+b_1(t)x''+b_2(t)x'+b_3(t)x=0,\ t\in I$ where b_1, b_2 and b_3 are (6)

functions defined and continuous on an interval *I*.

4. Let $A(t)$ be an $n\times n$ that is continuous in t on a closed and bounded interval t. Then prove that there exists a unique solution to the Initial value problem $x' = A(t)x, \quad x(t_o) = x_o; \ (t, t_o \in I)$ on I. (6)

5. Show that the matrix $\Phi(t) = \begin{bmatrix} e^{-3t} & te^{-3t} & e^{-3t}t^2/2! \\ 0 & e^{-3t} & te^{-3t} \\ 0 & 0 & e^{-3t} \end{bmatrix}$ is fundamental. (6)

6. State and Prove Picard's theorem (6)

7. Consider the IVP $x' = x^2 + \cos^2 t, \ x(0) = 0.$ Determine the largest interval of existence of its solution. (6)

8. Prove that the solution of the IVP $x'(t) = ax(t) + bx(t-r), \quad 0 \le t_0 \le t < \infty$ and $x(s) = \phi(s), \ t_0 - r \le s \le t_0$ is bounded if $\int_{t_0-r}^{t} \phi^2(s)ds \le \infty, \ t \ge t_0$ (6)

9. Show that the solution of the equation $x'(t) + 3x(t-1) = 0$ are oscillatory. (6)

--- End of Paper ---