QP Code: RD19MSC041 Reg. No | | | | | | | | | | AR 19

GIET UNIVERSITY, GUNUPUR – 765022

M. Sc (Third Semester) Examinations, December' 2020

CC 301 / MTPC301– FUNCTIONAL ANALYSIS –I (MATHEMATICS)

Time: 2hrs Maximum: 50 Marks

(The figures in the right hand margin indicate marks.)

Q.1. Answer **ALL** the questions (2 x 10 = 20)

a. Give two examples of a normed linear space.

b. Define bounded linear map.

c. Show that as a subspace of l^{∞}, the space c_{oo} is not closed.

d. Define support functional.

e. State Banach-Steinhaus Theorem.

f. State Open Mapping Theorem.

g. If X is a normed space and $A, B \in BL(X)$, then show that $\|AB\| \leq \|A\|\|B\|$.

h. Define transpose of a bounded linear from a normed space X into a normed space Y.

i. Define (i) eigen spectrum (ii) eigen value on a normed space X.

j. Let X be a normed space and X_0 be a dense subspace of X. For $x^{'} \in X^{'}$, let $F(x^{'})$ denote the restriction of x' to X_0. Show that F is a linear isometry from X' onto $X_0^{'}$.

PART – B (6 x 5=30 Marks)

<u>Answer ***ANY FIVE*** questions</u> Marks

2. Show that for $1 \leq p < \infty$, the set of scalar sequences l^{∞} is a normed space with respect to the norm $\|x\|_p = (|x_1|^p + |x_2|^p + \ldots)^{1/p}$ 6
3. Show that a linear map on a linear space X may be continuous with respect to some norm on X but discontinuous with respect to another norm on X. 6
4. State and prove Hahn-Banach separation theorem. 6
5. Let X be a normed space over a field K with dual X' and $f \in X'$ with $f \neq 0$. Let $a \in X$ with $f(a) = 1$ and $r > 0$. Prove that $U(a,r) \cap Z(f) = \emptyset$ if and only if $\|f\| \leq \frac{1}{r}$. 6
6. Let X and Y be normed spaces. If Z is a closed subspace of X, then show that the quotient map $Q: X \longrightarrow X/Z$ is continuous and open. 6
7. State and prove Closed Graph Theorem. 6
8. Let X be a Banach space. Show that $A \in BL(X)$ is invertible if and only if A is bijective. 6
9. If $1 \leq p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$, then show that the dual space of c_{00} with respect to the norm $\|.\|_p$ is linearly isometric to l^q. 6

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