QPC: AR 19 Reg. No

GIET UNIVERSITY, GUNUPUR – 765022

B. Tech (Fourth Semester - Regular) Examinations, April / May - 2021

Sub. Code -

Sub. Name: Thermodynamics, Heat and Mass transfer

Agricultural Engineering

Time: 3 hrs Maximum: 70 Marks

Answer ALL Questions

The figures in the right hand margin indicate marks.

PART – A: (Multiple Choice Questions) **(1 x 10 = 10 Marks)**

Q.1. Answer *ALL* questions					Ans
a.	An open system is one in which				iii
	(i)	Mass does not cross boundaries of the system, through energy may do so	(ii)	Neither mass nor energy crosses the boundaries of the system	
	(iii)	Both energy and mass crosses the boundary of the system	(iv)	Mass crosses the boundary the boundary but not energy	
b.	Properties of substances like pressure, temperature and density in thermodynamic coordinates are				
	(i)	Path function	(ii)	Point function	ii
	(iii)	Cyclic function	(iv)	Real function	
c.	Zeroth law of thermodynamics				iii
	(i)	Deals with conversion of mass and energy	(ii)	Deals with reversibility and irreversibility of process	
	(iii)	States that if two systems are both in equilibrium with a third system, they are in thermal equilibrium with each other	(iv)	Deals with heat engines	
d.	A heat exchange process in which the product of pressure and volume remains constant is known as				iv
	(i)	Heat exchange process	(ii)	Throttling process	
	(iii)	Isentropic process	(iv)	Hyperbolic process	
e.	Kelvin Planck's law deals with				iii
	(i)	Conservation of heat	(ii)	Conservation of work	
	(ii)	Conversion of heat into work	(iii)	Conversion of work into heat	
f.	The enthalpy of dry saturated steam ____________ with the increase of pressure				i
	(i)	Decreases	(ii)	Increases	
	(iii)	Remains constant	(iv)	All of the above	
g.	Superheated vapour behaves like				i
	(i)	Exactly as gas	(ii)	Liquid	
	(iii)	Ordinary vapour	(iv)	Approximately as a gas	
h.	In free convection heat transfer, Nusselt number is function of				i

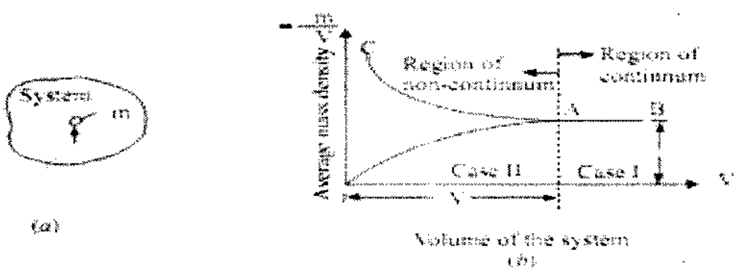
	(i) Grashoff number and Reynold number	(ii) Prandtl number and Reynold number	
	(iii) Grashoff number and Prandtl number	(iv) Grashoff number, Reynold number and Prandtl number	
i.	The critical radius is the insulation radius at which the resistance to heat flow is		i
	(i) Maximum	(ii) Minimum	
	(iii) Zero	(iv) None of these	
j.	The ratio of emissive power and absorptive power of all bodies is the same and is equal to the emissive power of a perfectly black body. This statement is known as		
	(i) Kirchoff's law	(ii) Stefan's law	i
	(iii) Wien's law	(iv) Planck's law	

PART – B: (Short Answer Questions) **(2 x 10 = 20 Marks)**

Q.2. Answer *ALL* questions	
a.	What is meant by thermodynamic system? How do you classify it? Thermodynamic system is defined as the any space or matter or group of matter where the energy transfer or energy conversions are studied. It may be classified into three types. (a) Open system (b) Closed system (c) Isolated system
b.	Define Intensive and Extensive properties. The properties which are independent on the mass of the system is called intensive properties. e.g., Pressure, Temperature, Specific Volume etc., The properties which are dependent on the mass of the system is called extensive properties. e.g., Total energy, Total volume, weight etc.
c.	Define: Specific heat capacity at constant pressure. It is defined as the amount of heat energy required to raise or lower the temperature of unit mass of the substance through one degree when the pressure kept constant. It is denoted by Cp.
d.	What is meant by reversible and irreversible process? A process is said to be reversible, it should trace the same path in the reverse direction when the process is reversed. It is possible only when the system passes through a continuous series of equilibrium state. If a system does not pass through continuous equilibrium state, then the process is said to be irreversible.
e.	State the Kelvin – Plank statement of second law of thermodynamics Kelvin – Plank states that it is impossible to construct a heat engine working on cyclic process, whose only purpose is to convert all the heat energy given to it into an equal amount of work.
f.	Define triple point. The triple point of a substance is the temperature and pressure at which the three phases of that substance coexist in thermodynamic equilibrium. It is that temperature and pressure at which the sublimation curve, fusion curve and the vaporisation curve meet.
g.	State Fourier's law of heat conduction. The law of heat conduction, also known as Fourier's law, states that the rate of heat transfer through a material is proportional to the negative gradient in the temperature and to the area, at right angles to that gradient, through which the heat flows.
h.	Distinguish between a black body and gray body.

	A black body is an object *that absorbs all the radiant energy reaching its surface* (for a black body $\alpha = 1$, $\rho = 0$, $\tau = 0$). No actual body is perfectly black; the concept of a black body is an idealization with which the radiation characteristics of real bodies can be conveniently compared. **Gray body:** If the radiative properties, α, ρ, τ of a body are assumed to be uniform over the entire wavelength spectrum, then such a body is called *gray body.* A *gray body* is also defined as one *whose absorptivity of a surface does not vary with temperature and wavelength of the incident radiation* [$\alpha = (\alpha)_\lambda$ = constant.].
i.	**Define LMTD of a heat exchanger** **Logarithmic mean temperature difference (LMTD)** *is defined as* that *temperature difference which, if constant, would give the same rate of heat transfer as actually occurs under variable conditions of temperature difference.*
j.	**State Fick's law of diffusion** **Fick's law of diffusion, which states that the mass flux of a constituent per unit area is proportional to the concentration gradient.**

PART – C: (Long Answer Questions) **(10 x 4 = 40 Marks)**

<u>Answer *ALL* questions</u>		**Mark s**
3. **a.**	**Explain what do you understand by concept of Continuum.** **Even the simplification of matter into molecules, atoms, electrons, etc. is too complex a picture for many problems of thermodynamics. Thermodynamics doesn't make any hypotheses about the structure of the matter of system. The volumes of system considered are quite large as compared to molecular dimensions. The system can be regarded as continuum. The system is supposed to contain continuous distribution of matter. There are no voids and cavities present. The pressure, temperature, density and other properties are average values of action of several molecules and atoms. This kind of idealization is a must for solving most of the problems. The laws and concepts of thermodynamics are not dependent of structure of matter.** **In accordance to this concept there is minimum limit of volume up to which the property remains continuum. Below this volume, there is sudden change in the value of the property. This type of region is called as region of discrete particles and the region for which the property are maintain is called as region of continuum. The volume up to which continuum properties are maintained is called as continuum limit.** $\rho = \lim_{\delta v \to \delta v'} [\delta m / \delta v]$  	**5**
b.	**Air enters a compressor at 10^5 Ka and 25° C having volume of 1.8 m^3/kg and is compressed to 5 x 10^5 Pa isothermally. Determine: (i) Work done (ii) Change in internal energy (iii) Heat transferred.**	**5**

Solution. Initial pressure of air, $p_1 = 10^5$ Pa

Initial temperature of air, $T_1 = 25 + 273 = 298$ K

Final pressure of air, $p_2 = 5 \times 10^5$ Pa

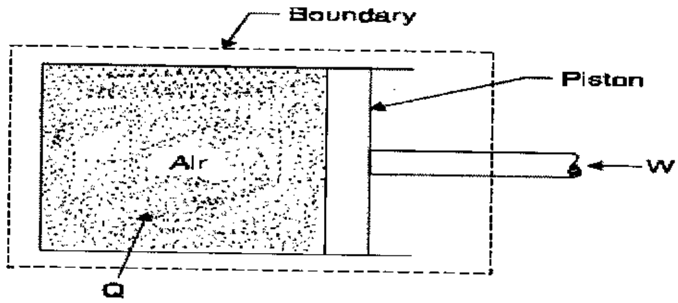
Final temperature of air, $T_2 = T_1 = 298$ K (isothermal process)

Since, it is a closed steady state process, we can write down the first law of thermodynamics

$$Q = (u_2 - u_1) + W \text{ ......per kg}$$

(i) For isothermal process :

$$W_{1-2} = \int_1^2 p \cdot dv = p_1 v_1 \log_e \left(\frac{p_1}{p_2}\right)$$



$p_1 v_1 = p_2 v_2$ for isothermal process

$$\therefore \quad W_{1-2} = -10^5 \times 1.8 \log_e \left(\frac{1 \times 10^5}{5 \times 10^5}\right)$$

$$= -2.897 \times 10^5 = -289.7 \text{ kJ/kg.}$$

(– ve sign indicates that the work is supplied to the air)

∴ **Work done on the air = 289.7 kJ/kg. (Ans.)**

(ii) Since temperature is constant,

$$\therefore \quad u_2 - u_1 = 0$$

∴ **Change in internal energy = zero. (Ans.)**

(iii) Again, $Q_{1-2} = (u_2 - u_1) + W$

$$= 0 + (-289.7) = -289.7 \text{ kJ}$$

(– ve sign indicates that heat is lost from the system to the surroundings)

∴ **Heat rejected = 289.7 kJ/kg. (Ans.)**

(OR)

c. **Derive steady flow energy equation.** **5**

Steady Flow Energy Equation (S.F.E.E.)

In many practical problems, the rate at which the fluid flows through a machine or piece of apparatus is constant. This type of flow is called *steady flow*.

Assumptions :

The following *assumptions* are made in the system analysis :

(i) The mass flow through the system remains constant.

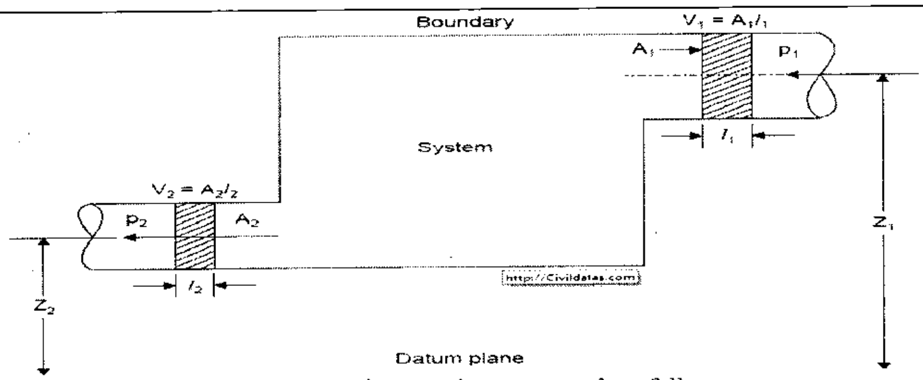
(ii) Fluid is uniform in composition.

(iii) The only interaction between the system and surroundings are work and heat.

(iv) The state of fluid at any point remains constant with time.

(v) In the analysis only potential, kinetic and flow energies are considered.

Fig. shows a schematic flow process for an open system. An open system is one in which both mass and energy may cross the boundaries. A wide interchange of energy may take place within an open system. Let the system be an automatic engine with the inlet manifold at the first state point and exhaust pipe as the second point. There would be an interchange of chemical energy in the fuel, kinetic energy of moving particles, internal energy of gas and heat transferred and shaft work within the system. From Fig. it is obvious that if there is no variation of flow of mass or energy with time across the boundaries of the system the steady flow will prevail. The conditions may pass through the cyclic or non-cyclic changes within the system. As a result the mass entering the system equals the mass leaving, also energy entering the system equals energy leaving.



The steady flow equation can be expressed as follows :

$$u_1 + \frac{C_1^2}{2} + Z_1g + p_1v_1 + Q = u_2 + \frac{C_2^2}{2} + Z_2g + p_2v_2 + W$$

$$(u_1 + p_1v_1) + \frac{C_1^2}{2} + Z_1g + Q = (u_2 + p_2v_2) + \frac{C_2^2}{2} + Z_2g + W$$

$$h_1 + \frac{C_1^2}{2} + Z_1g + Q = h_2 + \frac{C_2^2}{2} + Z_2g + W \qquad [\because$$

If Z_1 and Z_2 are neglected, we get

$$h_1 + \frac{C_1^2}{2} + Q = h_2 + \frac{C_2^2}{2} + W$$

where, Q = Heat supplied (or entering the boundary) per kg of fluid,
W = Work done by (or work coming out of the boundary) 1 kg of fluid,

C = Velocity of fluid ,
Z = Height above datum,
p = Pressure of the fluid,
u = Internal energy per kg of fluid, and
pv = Energy required for 1 kg of fluid.

This equation is applicable to any medium in any steady flow. It is applicable not only to rotary machines such as centrifugal fans, pumps and compressors but also to reciprocating machines such as steam engines.

In a steady flow the rate of mass flow of fluid at any section is the same as at any other section. Consider any section of cross-sectional area A, where the fluid velocity is C, the rate of volume flow past the section is CA. Also, since mass flow is volume flow divided by specific volume,

$$\therefore \qquad \text{Mass flow rate, } \dot{m} = \frac{CA}{v}$$

(where v = *Specific volume* at the section)

This equation is known as the **continuity of mass equation**.

d. **10 kg of fluid per minute goes through a reversible steady flow process. The properties of fluid at the inlet are: p_1 = 1.5 bar, ρ_1 = 26 kg/m^3, C_1 = 110 m/s and u_1 = 910 kJ/kg and at the exit are p_2 = 5.5 bar, ρ_2= 5.5 kg/m^3, C_2 = 190 m/s and u_2 = 710 kJ/kg. During the passage, the fluid rejects 55 kJ/s and rises through 55 metres. Determine: (i) the change in enthalpy and (ii) Work done during the process.** **5**

Solution. Flow of fluid = 10 kg/min

Properties of fluid at the **inlet** :

Pressure, p_1 = 1.5 bar = 1.5 × 10^5 N/m^2

Density, ρ_1 = 26 kg/m^3

Velocity, C_1 = 110 m/s

Internal energy, u_1 = 910 kJ/kg

Properties of the fluid at the **exit** :

Pressure, p_2 = 5.5 bar = 5.5 × 10^5 N/m^2

Density, ρ_2 = 5.5 kg/m^3

Velocity, C_2 = 190 m/s

Internal energy, u_2 = 710 kJ/kg

Heat *rejected* by the fluid,

Q = 55 kJ/s

Rise is elevation of fluid = 55 m.

(*i*) The change in enthalpy,

$$\Delta h = \Delta u + \Delta(pv)$$

$$\Delta(pv) = \frac{p_2v_2 - p_1v_1}{1}$$

$$= \frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} = \frac{5.5 \times 10^5}{5.5} - \frac{1.5 \times 10^5}{26}$$

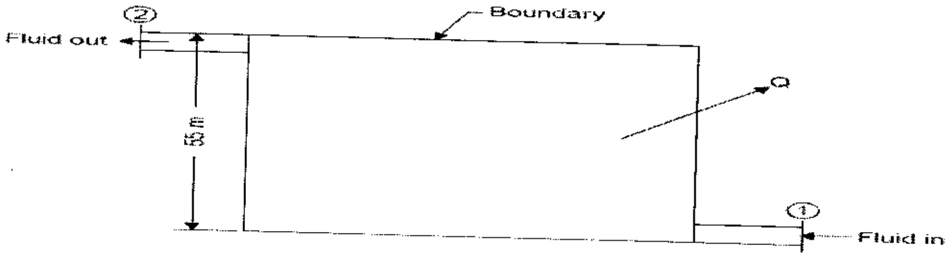
$$= 1 \times 10^5 - 0.0577 \times 10^5$$

$$= 10^5 \times 0.9423 \text{ Nm or J} = 94.23 \text{ kJ}$$

$$\Delta u = u_2 - u_1 = (710 - 910) = -200 \text{ kJ/kg}$$

Substituting the value in eqn. (*i*), we get

$$\Delta h = -200 + 94.23 = -\mathbf{105.77\ kJ/kg.}\quad \textbf{(Ans.)}$$



(*ii*) The steady flow equation for unit mass flow can be written as

$$Q = \Delta KE + \Delta PE + \Delta h + W$$

where Q is the heat transfer per kg of fluid

$$Q = 55 \text{ kJ/s} = \frac{55 \text{ kJ/s}}{\frac{10}{60} \text{ kg/s}} = 55 \times 6 = 330 \text{ kJ/kg}$$

Now,

$$\Delta KE = \frac{C_2^2 - C_1^2}{2} = \frac{(190)^2 - (110)^2}{2} \text{ Nm or J} = 12000 \text{ J or } 12 \text{ kJ/kg}$$

$$\Delta PE = (Z_2 - Z_1)\, g = (55 - 0) \times 9.81 \text{ Nm or J} = 539.5 \text{ J or} \approx 0.54 \text{ kJ/kg}$$

Substituting the value in steady flow equation,

$$-330 = 12 + 0.54 - 105.77 + W \quad \text{or} \quad W = -236.77 \text{ kJ/kg.}$$

Work done per second $= -236.77 \times \frac{10}{60} = -39.46$ kJ/s $= -$ **39.46 kW.** **(Ans.)**

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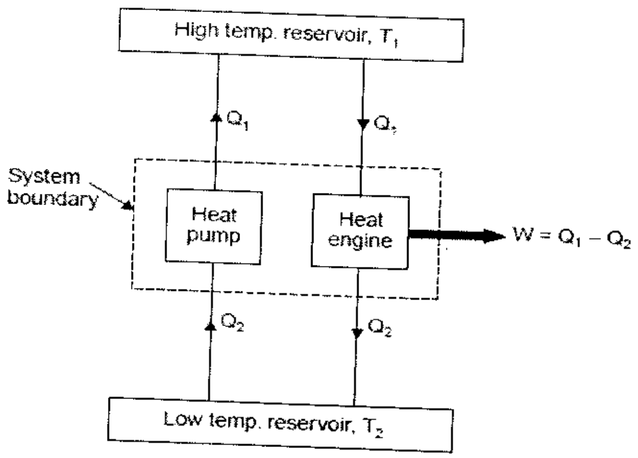
4. a. **Establish the equivalence of Kelvin-Planck and Clausius statements.** — 5

Equivalence of Clausius Statement to the Kelvin-Planck Statement

Consider a higher temperature reservoir T_1 and low temperature reservoir T_2. Fig. shows a heat pump which requires no work and transfers an amount of Q_2 from a low temperature to a higher temperature reservoir (in violation of the Clausius statement). Let an amount of heat Q_1 (greater than Q_2) be transferred from high temperature reservoir to heat engine which devolops a net work, $W = Q_1 - Q_2$ and rejects Q_2 to the low temperature reservoir. Since there is no heat interaction with the low temperature, it can be eliminated. The combined system of the heat engine and heat pump acts then like a heat engine exchanging heat with a single reservoir, which is the violation of the Kelvin-Planck statement.



b. **A heat engine receives heat at the rate of 1500 kJ/min and gives an output of 8.2 kW. Determine (i) The thermal efficiency (ii) the rate of heat rejection.** **5**

Solution. Heat received by the heat engine,

$$Q_1 = 1500 \text{ kJ/min} = \frac{1500}{60} = 25 \text{ kJ/s}$$

Work output, $W = 8.2 \text{ kW} = 8.2 \text{ kJ/s}$.

(*i*) Thermal efficiency, $\eta_{th} = \dfrac{W}{Q_1}$

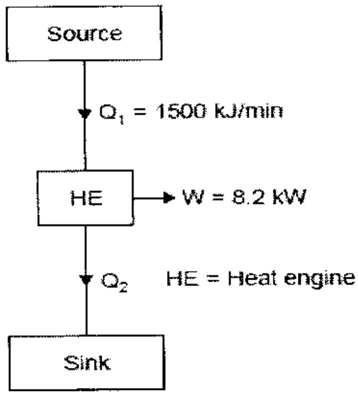
$$= \frac{8.2}{25} = 0.328 = 32.8\%$$

Hence, **thermal efficiency = 32.8%. (Ans.)**

(*ii*) Rate of heat rejection,

$$Q_2 = Q_1 - W = 25 - 8.2 = 16.8 \text{ kJ/s}$$

Hence, **the rate of heat rejection = 16.8 kJ/s. (Ans.)**



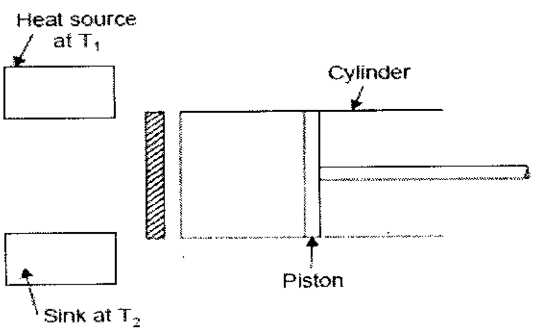
(OR)

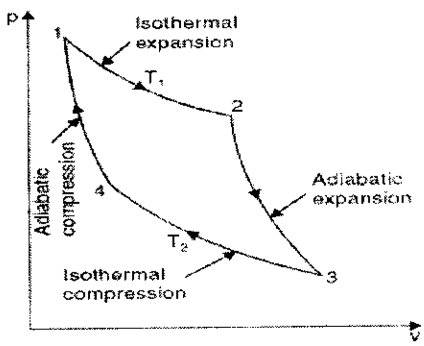
c. **Explain the working of Carnot cycle and derive the expression for its Thermal efficiency.** **5**

5.8. CARNOT CYCLE

The cycle was first suggested by a French engineer Sadi Carnot in 1824 which works on reversible cycle and is known as *Carnot cycle*.

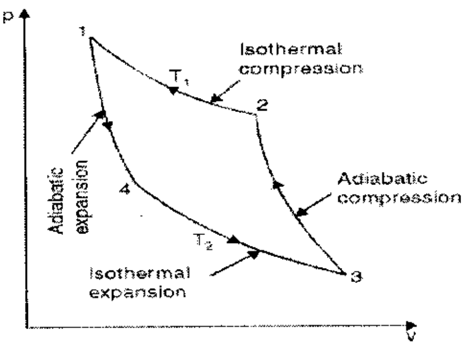
Any fluid may be used to operate the Carnot cycle . . . . . which is performed in an engine cylinder the head of which is supposed alternatively to be perfect conductor or a perfect insulator of a heat. Heat is caused to flow into the cylinder by the application of high temperature energy source to the cylinder head during expansion, and to flow from the cylinder by the application of a lower temperature energy source to the head during compression.





(*b*)

Carnot engine cycle



(*c*)

Carnot heat pump cycle

The *assumptions* made for describing the working of the Carnot engine are as follows :

(*i*) The piston moving in a cylinder does not develop any friction during motion.

(*ii*) The walls of piston and cylinder are considered as perfect insulators of heat.

(*iii*) The cylinder head is so arranged that it can be a perfect heat conductor or perfect heat insulator.

(*iv*) The transfer of heat does not affect the temperature of source or sink.

(*v*) Working medium is a perfect gas and has constant specific heat.

(*vi*) Compression and expansion are reversible.

Following are the *four stages* of Carnot cycle :

Stage 1. (Process 1-2). Hot energy source is applied. Heat Q_1 is taken in whilst the fluid expands isothermally and reversibly at constant high temperature T_1.

Stage 2. (Process 2-3). The cylinder becomes a perfect insulator so that no heat flow takes place. The fluid expands adiabatically and reversibly whilst temperature falls from T_1 to T_2.

Stage 3. (Process 3-4). Cold energy source is applied. Heat Q_2 flows from the fluid whilst it is compressed isothermally and reversibly at constant lower temperature T_2.

Stage 4. (Process 4-1). Cylinder head becomes a perfect insulator so that no heat flow occurs. The compression is continued adiabatically and reversibly during which temperature is raised from T_2 to T_1.

The work delivered from the system during the cycle is represented by the enclosed area of the cycle. Again for a closed cycle, according to first law of the thermodynamics the work obtained is equal to the difference between the heat supplied by the source (Q_1) and the heat rejected to the sink (Q_2).

$$\therefore \quad W = Q_1 - Q_2$$

Also, thermal efficiency, $$\eta_{th} = \frac{\text{Work done}}{\text{Heat supplied by the source}} = \frac{Q_1 - Q_2}{Q_1}$$

$$= 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \qquad \left[\because \begin{array}{l} Q_1 = m\,c_p\,T_1 \\ Q_2 = m\,c_p\,T_2 \\ \text{where, } m = \text{mass of fluid.} \end{array}\right]$$

Such an engine since it consists entirely of reversible processes, can operate in the reverse direction so that it follows the cycle shown in Fig. 5.5 (*b*) and operates as a **heat pump.** Q_2 is being taken in at the lower temperature T_2 during the isothermal expansion (process 4-3) and heat Q_1 is being rejected at the upper temperature T_1 (process 2-1). Work W will be needed to drive the pump. Again, the enclosed area represents this work which is exactly equal to that flowing from it when used as engine.

The Carnot cycle *cannot be performed in practice* because of the following **reasons** :

1. It is imposible to perform a frictionless process.

2. It is impossible to transfer the heat without temperature potential.

3. Isothermal process can be achieved only if the piston moves very slowly to allow heat transfer so that the temperature remains contant. Adiabatic process can be achieved only if the piston moves as fast as possible so that the heat transfer is negligible due to very short time available. The isothermal and adiabatic processes take place during the same stroke therefore the piston has to move very slowly for part of the stroke and it has to move very fast during remaining stroke. This variation of motion of the piston during the same stroke is not possible.

d. **3 kg of water at 80^0 C is mixed with 4 kg of water at 15^0 C in an isolated system. Calculate the change of entropy due to mixing process.** **5**

Applying first law of thermodynamics to the isolated system :

Total energy before mixing = Total energy after mixing

$$\therefore \quad 3c_{pw}(80 - 0) + 4c_{pw}(15 - 0) = 7\,c_{pw}(t_m - 0)$$

[c_{pw} = Specific heat of water at constant pressure]

or $$240c_{pw} + 60c_{pw} = 7c_{pw}\,t_m$$

or $$240 + 60 = 7\,t_m$$

$$\therefore \quad t_m = \frac{300}{7} = 42.85°C$$

Initial entropy of the system,

$$= 3c_{pw}\log_e\left(\frac{80+273}{273}\right) + 4c_{pw}\log_e\left(\frac{15+273}{273}\right)$$

$$= 0.7709c_{pw} + 0.2139\,c_{pw} = 0.9848\,c_{pw}$$

Final entropy of the system

$$= (3+4)\,c_{pw}\log_e\left(\frac{42.85+273}{273}\right) = 1.0205\,c_{pw}$$

Net change in entropy,

$$\Delta S = \text{Final entropy} - \text{Initial entropy}$$
$$= 1.0205 c_{pw} - 0.9848\ c_{pw} = 0.0357\ c_{pw}$$
$$= 0.0357 \times 4.187 \text{ kJ/K} \qquad [\because\ c_{pw} = 4.187 \text{ kJ/kg K}]$$
$$= 0.1495 \text{ kJ/K}$$

Hence, **net change in entropy = 0.1495 kJ/K. (Ans.)**

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5. a. **Explain the following terms relating to steam formation. (i) Sensible heat (ii) Latent heat of evaporation (iii) Dryness fraction of steam (iv) Enthalpy of wet steam (v) Enthalpy of superheated steam.** — **5**

1. Sensible heat of water (h_f). *It is defined as the quantity of heat absorbed by 1 kg of water when it is heated from 0°C (freezing point) to boiling point.* It is also called total heat (or enthalpy) of water or liquid heat invariably. It is reckoned from 0°C where sensible heat is taken as zero. If 1 kg of water is heated from 0°C to 100°C the sensible heat added to it will be 4.18 × 100 = 418 kJ but if water is at say 20°C initially then sensible heat added will be 4.18 × (100 − 20) = 334.4 kJ. This type of heat is denoted by letter h_f and its value can be directly read from the steam tables.

Note. The value of specific heat of water may be taken as 4.18 kJ/kg K at low pressures but at high pressures it is different from this value.

2. Latent heat or hidden heat (h_{fg}). *It is the amount of heat required to convert water at a given temperature and pressure into steam at the same temperature and pressure.* It is expressed by the symbol h_{fg} and its value is available from steam tables. *The value of latent heat is not constant and varies according to pressure variation.*

3. Dryness fraction (x). The term dryness fraction is related with wet steam. *It is defined as the ratio of the mass of actual dry steam to the mass of steam containing it.* It is usually expressed by the symbol 'x' or 'q'.

If m_s = Mass of dry steam contained in steam considered, and

m_w = Weight of water particles in suspension in the steam considered,

Then,
$$x = \frac{m_s}{m_s + m_w} \qquad ...(3.2)$$

Thus if in 1 kg of wet steam 0.9 kg is the dry steam and 0.1 kg water particles then $x = 0.9$.

Note. No steam can be completely dry and saturated, so long as it is in contact with the water from which it is being formed.

4. Total heat or enthalpy of wet steam (h). *It is defined as the quantity of heat required to convert 1 kg of water at 0°C into wet steam at constant pressure.* It is the sum of total heat of water and the latent heat and this sum is also called **enthalpy**.

In other words,
$$h = h_f + x h_{fg} \qquad ...(3.3)$$

If steam is dry and saturated, then $x = 1$ and $h_g = h_f + h_{fg}$.

5. Superheated steam. When steam is heated after it has become dry and saturated, it is called superheated steam and the process of heating is called *superheating*. *Superheating is always carried out at constant pressure.* The additional amount of heat supplied to the steam during superheating is called as *'Heat of superheat'* and can be calculated by using the specific heat of superheated steam at constant pressure (c_{ps}), the value of which varies from 2.0 to 2.1 kJ/kg K depending upon pressure and temperature.

b. **What amount of heat would be required to produce 4.4 kg of steam at a pressure of 6 bar and temperature of 250^{0}C from water at 30^{0}C? Take specific heat for superheated steam as 2.2 kJ/kg K** — **5**

Solution. Mass of steam to be produced, $m = 4.4$ kg

Pressure of steam, $p = 6$ bar

Temperature of steam, $t_{sup} = 250°C$

Temperature of water $= 30°C$

Specific heat of steam, $c_{ps} = 2.2$ kJ/kg

At 6 bar, 250°C : From steam tables,

$$t_s = 158.8°C,\ h_f = 670.4 \text{ kJ/kg},\ h_{fg} = 2085 \text{ kJ/kg}$$

Enthalpy of 1 kg superheated steam reckoned from 0°C,

$$h_{sup} = h_f + h_{fg} + c_{ps}(T_{sup} - T_s)$$
$$= 670.4 + 2085 + 2.2(250 - 158.8)$$
$$= 2956 \text{ kJ}$$

Amount of heat already with 1 kg of water

$$= 1 \times 4.18 \times (30 - 0) = 125.4 \text{ kJ}$$

Net amount of heat required to be supplied per kg

= 2956 – 12

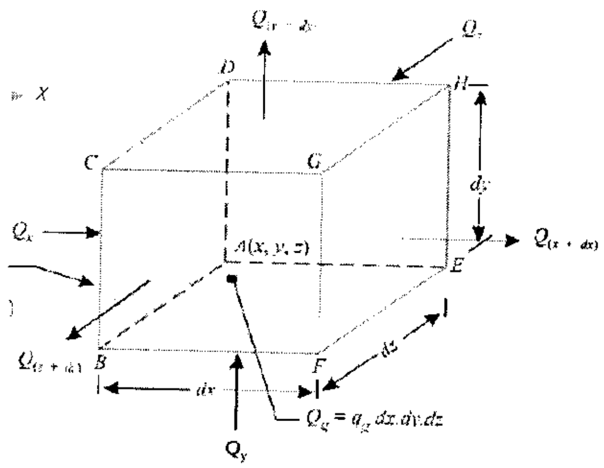
Total amount of heat required

$= 4.4 \times 2830.6 =$ **12454.6 kJ. (Ans.)**

(OR)

c. **Derive general Heat conduction equation in Cartesian co-ordinates.** 5

Consider an infinitesimal rectangular parallelopiped (volume element) of sides *dx*, *dy* and *dz* parallel, respectively, to the three axes (X, Y, Z) in a medium in which temperature is varying with location and time as shown in Fig. 2.1.



Let, t = Temperature at the left face *ABCD*; this temperature may be assumed uniform over the entire surface, since the area of this face can be made arbitrarily *small*, and

$\frac{dt}{dx}$ = Temperature changes and rate of change along *X*-direction.

Then, $\left(\frac{\partial t}{\partial x}\right)dx$ = Change of temperature through distance *dx*, and

$t + \left(\frac{\partial t}{\partial x}\right)dx$ = temperature on the right face *EFGH* (at a distance *dx* from the left face *ABCD*).

Further, let, k_x, k_y, k_z = Thermal conductivities (direction characteristics of the material) along *X*, *Y* and *Z* axes.

[If the directional characterisitics of a material are equal/same, it is called an "*Isotropic material*" and if unequal/different "*Anisotropic material*".]

q_g = Heat generated per unit volume per unit time.

[Inside the control volume there may be heat sources due to flow of electric current in electric motors and generators, nuclear fission etc.]

Energy balance/equation for volume element :

Net heat accumulated in the element due to conduction of heat from all the coordinate directions considered (A) + heat generated within the element (B) = Energy stored in the element (C). ...(1)

Let, Q = Rate of heat flow in a direction, and

$Q' = (Q.d\tau)$ = Total heat flow (flux) in that direction (in time $d\tau$).

A. *Net heat accumulated in the element due to conduction of heat from all the directions considered:*

Quantity of heat flowing into the element from the left face $ABCD$ during the time interval $d\tau$ in X-direction is given by :

Heat influx,
$$Q'_x = -k_x\,(dy.dz)\,\frac{\partial t}{\partial x}\cdot d\tau \qquad ...(i)$$

During the same time interval $d\tau$ the heat flowing out of the right face of control volume ($EFGH$) will be :

Heat efflux,
$$Q'_{(x+dx)} = Q'_x + \frac{\partial}{\partial x}(Q'_x)\,dx \qquad ...(ii)$$

$\therefore$ Heat accumulation in the element due to heat flow in X-direction,

$$dQ'_x = Q'_x - \left[Q'_x + \frac{\partial}{\partial x}(Q'_x)\,dx\right] \qquad \text{[Subtracting } (ii) \text{ from } (i)]$$
$$= -\frac{\partial}{\partial x}(Q'_x)\,dx$$
$$= -\frac{\partial}{\partial x}\left[-k_x\,(dy.dz)\,\frac{\partial t}{\partial x}\,.\,d\tau\right]dx$$
$$= \frac{\partial}{\partial x}\left[k_x\,\frac{\partial t}{\partial x}\right]dx.dy.dz.d\tau$$

Similarly the heat accumulated due to heat flow by conduction along Y and Z directions in time $d\tau$ will be :

$$dQ'_y = \frac{\partial}{\partial y}\left[k_y\,\frac{\partial t}{\partial y}\right]dx.dy.dz.d\tau$$
$$dQ'_z = \frac{\partial}{\partial z}\left[k_z\,\frac{\partial t}{\partial z}\right]dx.dy.dz.d\tau$$

$\therefore$ *Net heat accumulated* in the element due to conduction of heat from all the coordinate directions considered

$$= \frac{\partial}{\partial x}\left[k_x\,\frac{\partial t}{\partial x}\right]dx.dy.dz.d\tau + \frac{\partial}{\partial y}\left[k_y\,\frac{\partial t}{\partial y}\right]dx.dy.dz.d\tau + \frac{\partial}{\partial z}\left[k_z\,\frac{\partial t}{\partial z}\right]dx.dy.dz.d\tau$$
$$= \left[\frac{\partial}{\partial x}\left(k_x\,\frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y\,\frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\,\frac{\partial t}{\partial z}\right)\right]dx.dy.dz.d\tau$$

B. *Total heat generated within the element* (Q_g') :

The total heat generated in the element is given by

$$Q_g' = q_g\,(dx.dy.dz)\,d\tau$$

C. *Energy stored in the element* :

The total heat accumulated in the element due to heat flow along coordinate axes , and the heat generated within the element together serve to increase the thermal energy of the element/lattice. This increase in thermal energy is given by

$$\rho\,(dx.dy.dz)\,c.\frac{\partial t}{\partial \tau}\,.\,d\tau$$

[$\because$ Heat stored in the body = Mass of the body × specific heat of the body material × rise in the temperature of body].

Now, substituting eqns. in the eqn. (1), we have

$$\left[\frac{\partial}{\partial x}\left(k_x\,\frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\,\frac{\partial t}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\,\frac{\partial t}{\partial z}\right)\right]dx.dy.dz.d\tau + q_g\,(dx.dy.dz.)d\tau = \rho\,(dx.dy.dz)\,c.\frac{\partial t}{\partial \tau}\,.\,d\tau$$

Dividing both sides by $dx.dy.dz.d\tau$, we have

$$\frac{\partial}{\partial x}\left(k_x\,\frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\,\frac{\partial t}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\,\frac{\partial t}{\partial z}\right) + q_g = \rho.c.\frac{\partial t}{\partial \tau}$$

or, using the vector operator ∇, we get

$$\nabla .(k\nabla t)+q_g = \rho.c.\frac{\partial t}{\partial \tau}$$

This is known as the **general heat conduction equation for 'non-homogeneous material', 'self heat generating'** and **'unsteady three-dimensional heat flow'**. This equation establishes in *differential form the relationship between the time and space variation of temperature at any point of solid through which heat flow by conduction takes place.*

d.	**An exterior wall of a house may be approximated by a 0.1 m layer of common brick (k = 0.7 W/m^{0}C) followed by a 0.04 layer of gypsum plaster (k = 0.48 W/m^{0}C). What thickness of loosely packed rock wool insulation (k = 0.065 W/m^{0}C) should be added to reduce the heat loss or gain through the wall by 80%?**	5

Thickness of common brick, $L_A = 0.1$ m

Thickness of gypsum plaster, $L_B = 0.04$m

Thickness of rock wool, $L_C = x$ (in m) = ?

Thermal conductivities :

Common brick, $k_A = 0.7$ W/m°C;

Gypsum plaster, $k_B = 0.48$ W/m°C;

Rock wool, $k_C = 0.065$ W/m°C.

Case I. *Rock wool insulation not used :*

$$Q_1 = \frac{A(\Delta t)}{\frac{L_A}{k_A}+\frac{L_B}{k_B}} = \frac{A(\Delta t)}{\frac{0.1}{0.7}+\frac{0.04}{0.48}} \quad ...(i)$$

Case II. *Rock wool insulation used :*

$$Q_2 = \frac{A(\Delta t)}{\frac{L_A}{k_A}+\frac{L_B}{k_B}+\frac{L_C}{k_C}} = \frac{A(\Delta t)}{\frac{0.1}{0.7}+\frac{0.04}{0.48}+\frac{x}{0.065}} \quad ...(ii)$$

But, $Q_2 = (1-0.8)\,Q_1 = 0.2\,Q_1$ ...(*given*)

$\therefore$
$$\frac{A(\Delta t)}{\frac{0.1}{0.7}+\frac{0.04}{0.48}+\frac{x}{0.065}} = 0.2\times\frac{A(\Delta t)}{\frac{0.1}{0.7}+\frac{0.04}{0.48}}$$

or,
$$\frac{0.1}{0.7}+\frac{0.04}{0.48} = 0.2\left[\frac{0.1}{0.7}+\frac{0.04}{0.48}+\frac{x}{0.065}\right]$$

or, $0.1428+0.0833 = 0.2\,[0.1428+0.0833+15.385x]$

or, $0.2261 = 0.2\,(0.2261+15.385\,x)$

or, $x = 0.0588$ m or 58.8 mm

Thus, the *thickness of rock wool insulation should be* **58.8 mm** **(Ans.)**

6.	**State and prove Kirchhoff's law of radiation.**	5
a.	The law states that *at any temperature the ratio of total emissive power E to the total absorptivity α is a constant for all substances which are in thermal equilibrium with their environment.*	

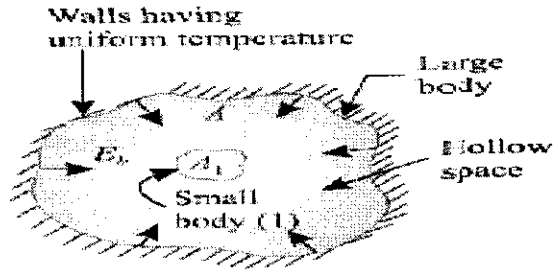


Fig. . Derivation of Kirchhoff's law.

Let us consider a large radiating body of surface area A which encloses a small body (1) of surface area A_1 (as shown in Fig. . . .). Let the energy fall on the unit surface of the body at the rate E_b. Of this energy, generally, a fraction α, will be absorbed by the small body. Thus, this energy absorbed by the small body (1) is $\alpha_1 A_1 E_b$, in which α_1 is the absorptivity of the body. When thermal equilibrium is attained, the *energy absorbed* by the body (1) must be equal to the *energy emitted*, say, E_1 per unit surface. Thus, at equilibrium, we may write

$$A_1E_1 = \alpha_1 A_1 E_b$$

Now we remove body (1) and replace it by body (2) having absorptivity α_2. The radiative energy impinging on the surface of this body is again E_b. In this case, we may write

$$A_2E_2 = \alpha_2 A_2 E_b$$

By considering generality of bodies, we obtain

$$E_b = \frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E}{\alpha}$$

Also, as per definition of emissivity ε, we have

$$\varepsilon = \frac{E}{E_b}$$

or, $$E_b = \frac{E}{\varepsilon}$$

By comparing eqns. . . . and . . . ., we obtain

$$\varepsilon = \alpha$$

(α is always smaller than 1. Therefore, the emissive power E is always smaller than the emissive power of a black body at equal temperature.)

Thus, Kirchhoff's law also states that *the emissivity of a body is equal to its absorptivity when the body remains in thermal equilibrium with its surroundings.*

b. **Calculate the following for an industrial furnace in the form of black body and emitting radiation at 2500^{0}C.**

(i) Monochromatic emissive power at 1.2 μm length. (ii) Wavelength at which the emission is maximum. (iii) maximum emissive power (iv) Total emissive power (v) Total emissive power of the furnace if it is assumed as a real surface with emissivity equal to 0.9. **5**

Solution. *Given* : $T = 2500 + 273 = 2773 K$; $\lambda = 1.2\ \mu m$, $\varepsilon = 0.9$

(*i*) **Monochromatic emissive power at 1.2 μm length, $(E_\lambda)_b$:**

According to Planck's law,

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right) - 1}$$

where, $C_1 = 3.742 \times 10^8$ W.μm⁴/m² $= 0.3742 \times 10^{-15}$ W.m⁴/m², and
$C_2 = 1.4388 \times 10^{-2}$ mK

Substituting the values, we get

$$(E_\lambda)_b = \frac{0.3742 \times 10^{-15} \times (1.2 \times 10^{-6})^{-5}}{\exp\left(\frac{1.4388 \times 10^{-2}}{1.2 \times 10^{-6} \times 2773}\right) - 1} = \frac{1.5 \times 10^{14}}{74.48} = \mathbf{2.014 \times 10^{12}\ W/m^2\ (Ans.)}$$

(*ii*) Wavelength at which the emission is maximum, λ_{max}:

Accodrding to Wien's displacement law,

$$\lambda_{max} = \frac{2898}{T} = \frac{2898}{2773} = \mathbf{1.045\ \mu m\ \ (Ans.)}$$

(*iii*) Maximum emissive power, $(E_{\lambda b})$max:

$$(E_{\lambda b})_{max} = 1.285 \times 10^{-5}\, T^5 \text{ W/m}^2 \text{ per metre length}$$
$$= 1.285 \times 10^{-5} \times (2773)^5 = \mathbf{2.1 \times 10^{12}\ W/m^2\ per\ metre\ length\ (Ans.)}$$

(*iv*) Total emissive power, E_b :

$$E_b = \sigma T^4 = 5.67 \times 10^{-8} (2773)^4 = 5.67 \left(\frac{2773}{100}\right)^4 = \mathbf{3.352 \times 10^6\ W/m^2.\ (Ans.)}$$

(*v*) Total emissive power, E with emissivity (ε) = 0.9 :

$$E = \varepsilon\, \sigma T^4 = 0.9 \times 5.67 \left(\frac{2773}{100}\right)^4 = \mathbf{3.017 \times 10^6\ W/m^2.\ (Ans.)}$$

(OR)

c. **Derive an expression for Logarithmic Mean Temperature Difference for parallel flow heat exchanger.** **5**

10.4.1. LOGARITHMIC MEAN TEMPERATURE DIFFERENCE FOR "PARALLEL FLOW"

Refer Fig. 10.9, which shows the flow arrangement and distribution of temperature in a single-pass parallel flow heat exchanger.

Let us consider an elementary area dA of the heat exchanger. The rate of flow of heat through this elementary area is given by

$$dQ = U\, dA\, (t_h - t_c) = U \,.\, dA \,.\, \Delta t$$

As a result of heat transfer dQ through the area dA, the hot fluid is cooled by dt_h whereas the cold fluid is heated up by dt_c. The energy balance over a differential area dA may be written as

$$dQ = -\, \dot{m}_h \,.\, c_{ph} \,.\, dt_h = \dot{m}_c \,.\, c_{pc} \,.\, dt_c = U \,.\, dA \,.\, (t_h - t_c)$$

(Here d_{th} is – ve and d_{tc} is + ve)

or, $$dt_h = -\frac{dQ}{\dot{m}_h\, c_{ph}} = -\frac{dQ}{C_h}$$

and, $$dt_c = \frac{dQ}{\dot{m}_c\, c_{pc}} = \frac{dQ}{C_c}$$

where, $C_h = \dot{m}_h\ c_{ph}$ = Heat capacity or water equivalent of hot fluid, and
$C_c = \dot{m}_c\ c_{pc}$ = Heat capacity or water equivalent of cold fluid.

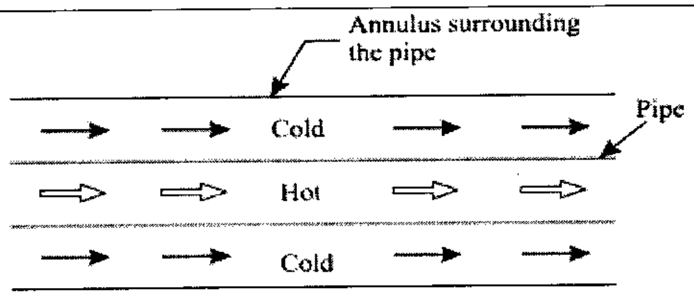
$\dot{m}_h$ and $\dot{m}_c$ are the mass flow rates of fluids and c_{ph} and c_{pc} are the respective specific heats.

$$\therefore \quad dt_h - dt_c = -\, dQ \left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

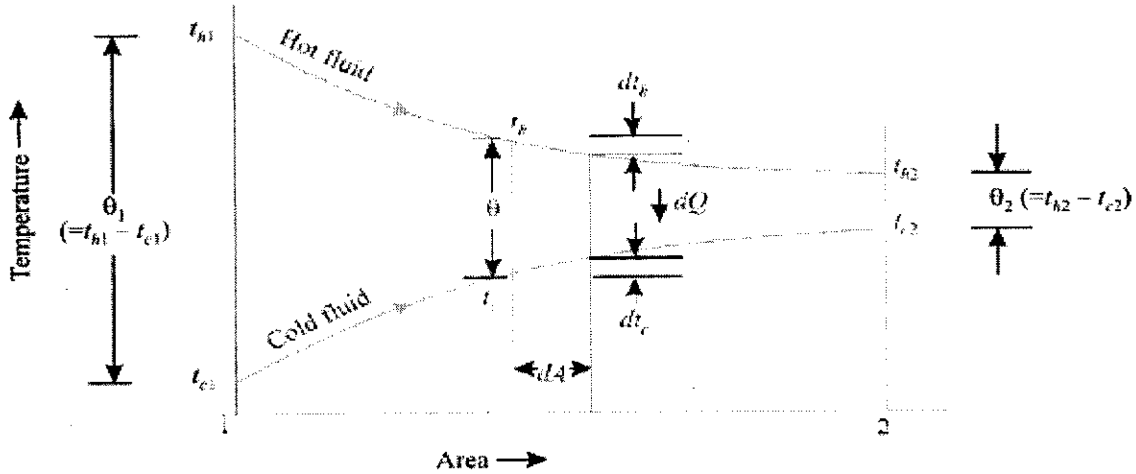
$$d\theta = -\, dQ \left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Substituting the value of dQ from eqn. (10.4) the above equation becomes

$$d\theta = -\, U \,.\, dA\, (t_h - t_c) \left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$



(a) Flow arrangement



or, $$d\theta = -U \cdot dA \cdot \theta \left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

or, $$\frac{d\theta}{\theta} = -U \cdot dA \left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Integrating between inlet and outlet conditions (*i.e.* from $A = 0$ to $A = A$), we get

$$\int_1^2 \frac{d\theta}{\theta} = -\left[\frac{1}{C_h} + \frac{1}{C_c}\right] \int_{A=0}^{A=A} U \cdot dA$$

or, $$ln\,(\theta_2/\theta_1) = -UA\left[\frac{1}{C_h} + \frac{1}{C_c}\right]$$

Now, the total heat transfer rate between the two fluids is given by

$$Q = C_h\,(t_{h1} - t_{h2}) = C_c\,(t_{c2} - t_{c1})$$

or, $$\frac{1}{C_h} = \frac{t_{h1} - t_{h2}}{Q}$$

$$\frac{1}{C_c} = \frac{t_{c2} - t_{c1}}{Q}$$

Substituting the values of $\frac{1}{C_h}$ and $\frac{1}{C_c}$ into eqn. , we get

$$ln\,(\theta_2/\theta_1) = -UA\left[\frac{t_{h1} - t_{h2}}{Q} + \frac{t_{c2} - t_{c1}}{Q}\right]$$

$$= \frac{UA}{Q}\,[(t_{h2} - t_{c2}) - (t_{h1} - t_{c1})] = \frac{UA}{Q}\,(\theta_2 - \theta_1)$$

$$Q = \frac{UA\,(\theta_2 - \theta_1)}{ln\,(\theta_2/\theta_1)}$$

The above equation may be written as

$$Q = U\,A\,\theta_m$$

where, $$\theta_m = \frac{\theta_2 - \theta_1}{ln\,(\theta_2/\theta_1)} = \frac{\theta_1 - \theta_2}{ln\,(\theta_1/\theta_2)}$$

θ_m is called the *logarithmic mean temperature difference* (*LMTD*).

d. **The flow rates of hot and cold water streams running through a parallel flow heat exchanger are 0.2 kg/s and 0.5 kg/s respectively. The inlet temperatures on the hot and cold sides are 75^0 C and 20^0 C respectively. The exit temperature of hot water is 45^0 C. If the individual heat transfer co-efficient on both sides are 650 W/m^2 K. Calculate the area of the heat exchanger.** 5

Solution. *Given* : $\dot{m}_h = 0.2$ kg/s; $\dot{m}_c = 0.5$ kg/s; $t_{h1} = 75°$ C; $t_{h2} = 45°$C; $t_{c1} = 20°$C; $h_i = h_o =$ 650 W/m^2°C.

The area of heat exchanger, *A* **:**

The heat exchanger is shown diagrammatically in Fig. 10.14.

The heat transfer rate,

$$Q = \dot{m}_h \times c_{ph} \times (t_{h1} - t_{h2})$$

$$= 0.2 \times 4.187 \times (75 - 45) = 25.122 \text{ kJ/s}$$

Heat lost by hot water = Heat gained by cold water

$$\dot{m}_h \times c_{ph} \times (t_{h1} - t_{h2}) = \dot{m}_c \times c_{pc} \times (t_{c2} - t_{c1})$$

$$0.2 \times 4.187 \times (75 - 45) = 0.5 \times 4.187 \times (t_{c2} - 20)$$

$$\therefore \quad t_{c2} = 32°\text{C}$$

Logarithmic mean temperature difference (*LMTD*) is given by

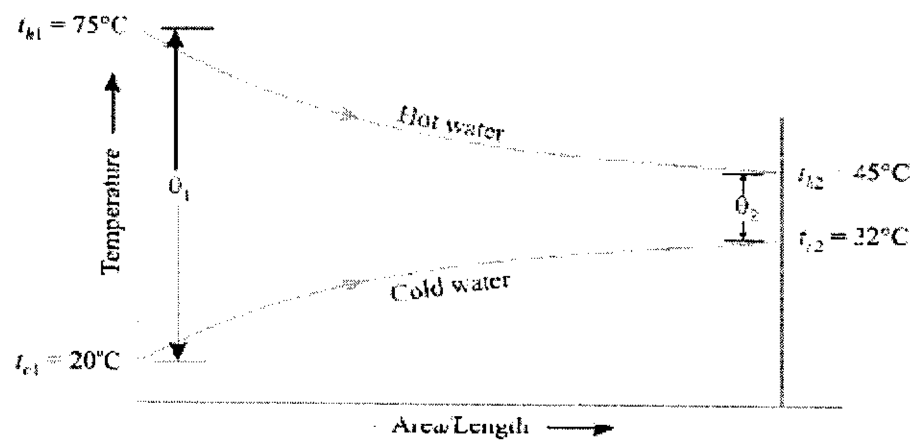
$$\theta_m = \frac{\theta_1 - \theta_2}{\ln(\theta_1/\theta_2)}$$

or,

$$\theta_m = \frac{(t_{h1} - t_{c1}) - (t_{h2} - t_{c2})}{\ln[(t_{h1} - t_{c1})/(t_{h2} - t_{c2})]}$$

$$= \frac{(75 - 20) - (45 - 32)}{\ln[(75 - 20)/(45 - 32)]}$$

$$= \frac{55 - 13}{\ln(55/13)} = 29.12°\text{C}$$



(*b*) Temperature distribution

Parallel flow heat exchanger.

Overall heat transfer coefficient *U* is calculated from the relation,

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

$$= \frac{1}{650} + \frac{1}{650} = \frac{1}{325}$$

$$\therefore \quad U = 325 \text{ W/m}^2\text{°C}$$

Also,

$$Q = U A \theta_m$$

or,

$$A = \frac{Q}{U\theta_m} = \frac{25.122 \times 1000}{325 \times 29.12} = \mathbf{2.66\ m^2} \qquad \textbf{(Ans.)}$$

--- End of Paper ---