Registration No. : | | | | | | | | | | |

Total number of printed pages – 3

B. Tech

FECS 6401

Seventh Semester Examination – 2011

INTRODUCTION TO DIGITAL SIGNAL PROCESSING

Full Marks – 70

Time : 3 - Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

(a) State whether the following system is linear or not $y(t) = 2 + x(t)$.

(b) State whether a unit ramp sequence is a power signal or an energy signal.

(c) State whether the following signal is periodic or not, if periodic, find the fundamental period : $x(n) = \cos\left(\frac{\pi}{3}n\right) + \sin\left(\frac{3\pi}{4}n\right)$.

(d) Find the z-transform of the signal $x(n) = u(-n)$. Also state the *ROC*.

(e) Write down the relationship between DFT and z-transform.

(f) What do you mean by twiddle factor and write down its two properties.

(g) How many complex additions and multiplications are required to compute 64-point DFT via decimation-in-frequency FFT algorithm ?

(h) Write down two main advantages of FIR filter over IIR filter.

(i) Why impulse-invariant method is not preferred in the design of IIR filter other than Low Pass Filter ?

(j) Give the bilinear transform equation between s-plane and z-plane.

P.T.O.

2. (a) What is the magnitude of an odd function at index $n = 0$. 2

(b) Derive a closed-form expression for the convolution of $x(n)$ and $h(n)$ where $x(n) = 2^n u(n)$ and $h(n) = (1/2)^n u(n)$. 3

(c) Consider the following DTS whose output $y(n)$ is related to the input $x(n)$ as $y(n) = x(n^2)\,u(n)$. Determine whether or not the above systems are (i) linear, (ii) time-invariant, (iii) static, (iv) stable, (v) causal. 5

3. (a) Give the Direct-form II structure of the system described by the difference equation given by $y(n) + \sum_{k=1}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$ for N > M. 4

(b) Determine the total solution of the difference equation

$$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$$ where the forcing function is

$$x(n) = \left(\frac{1}{2}\right)^n u(n).$$ 6

4. (a) Find the z-transform of the following signal $x(n) = \left(\frac{1}{2}\right)^n [u(n) - u(n-10)]$. 3

(b) Find the inverse z-transform of $X(z) = (1 + bz^{-1}) \log (1 + az^{-1})$, $|z| > |a|$. 7

5. (a) Prove that $DFT[x^*(n)] = X^*(N-k)$. 3

(b) Find the linear convolution $y(n) = x(n)^* h(n)$, where $x(n) = \{0.5, 2, -1.5, -1, 0, 0.75, 3, 2, 1.5, 1, -0.75, 2\}$ and $h(n) = \{1, 2, -1\}$ using overlap-add method. 7

6. (a) For the analog transfer function $H(s) = \frac{2}{(s+1)(s+2)}$, determine $H(z)$ using impulse invariance method. Assume $T = 1$ sec. 4

(b) Obtain the cascade and parallel realizations for the system function given by $H(z) = \frac{1 + \frac{1}{4}z^{-1}}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}\right)}$. 6

7. Prove the Decimation-In-Time Radix-2 FFT algorithm with neat Butterfly diagram (Consider N = 8). 10

8. (a) Write a short note on Gibbs phenomenon. 3

(b) Design an ideal Low Pass Filter with a frequency response

$$H_d(e^{j\omega}) = \begin{cases} 1, & -\frac{\pi}{2} \le \omega \le \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

Find the values of *h(n)* for N = 9. Also, find the frequency response. 7