Registration No. : | | | | | | | | | | |

Total number of printed pages – 3

B. Tech

BSCM 1205

Third Semester Regular Examination – 2014

MATHEMATICS – III

BRANCH(S) : AEIE, AERO, AUTO, BIOMED, BIOTECH, CHEM, CIVIL, CSE, EC, EEE, EIE, ELECTRICAL, ENV, ETC, FASHION, IEE, IT, MANUTECH, MECH, MINERAL, MINING, MM, MME, PLASTIC, TEXTILE

QUESTION CODE : H 371

Full Marks – 70

Time – 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2 × 10

   (a) What is the difference between a general solution and a complete solution ?

   (b) What is Dirichlet problem ?

   (c) Find the vibration of the string of length 1 and c = 1 starting with initial velocity zero and initial deflection $f(x) = k(x - x^3)$.

   (d) What do you know about types of Partial differential equations ? State the type of the equation $u_{xx} + 5u_{xy} + 2u_{yy} = 0$.

   (e) Find the fixed points of the mapping $w = \dfrac{5z + 4}{z + 5}$.

   (f) Find, whether f(z) is continous at z = 0 if $f(z) = \begin{cases} \dfrac{re(z^2)}{|z|^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$.

P.T.O.

(g) Define convergence, absolute convergence, conditional convergence of a series with giving example.

(h) Check whether the following sequence is bounded, convergent and find its limit point : $z_n = \frac{(1+i)^{2n}}{2^n}$.

(i) State Morera's theorem.

(j) Explain different types of singularity giving examples.

2. (a) Solve : $p^2q(x^2 + y^2) = p^2 + q$. 5

(b) Solve : $(x + pz)^2 + (y + qz)^2 = 1$. 5

3. (a) Solve : $4D^2 - 4DD' + D'^2 = 16 \log (x + 2y) + x^2$. 5

(b) Solve the heat equation when ends of the rod are insulated using variable separable method. 5

4. (a) Solve $u_{xx} + u_{yy} = 0$ which satisfies the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin(n\pi x/l)$. 5

(b) Find the vibrations of a rectangular membrane of sides a = 4 ft and b = 2 ft if the tension is 12.5 lb/ft, the density is 2.5 slugs/ft^2, the initial velocity is 0 and the initial displacement is $f(x, y) = 0.1(4x - x^2)(2y - y^2)$. 5

5. (a) Are the following functions analytic ? 5

(i) $f(z) = z^2 + \frac{1}{z^2}$

(ii) $f(z) = xy + ix^2y$

(b) Find the analytic function whose real part is $e^x \sin(x^2 - y^2)$. 5

6. (a) Evaluate the integral $\int_c (z+1)^2 \, dz$ where c is the boundary of the rectangle with vertices at the point 2 + 3i, – 2 + 3i, – 2 – 3i, 2 – 3i. 5

(b) Evaluate $\int_c \frac{2z^2 + 5}{(z+1)^3 (z^2 + 3)} \, dz$ where c is a circle having center at 2 and radius 5. 5

7. (a) Find the Taylor's series expansion of $f(z) = ze^{2z}$ around $z = -1$, find the radius of convergence of the above. 5

(b) Find the Laurent's series expansion of $f(z) = \frac{7z-2}{z^3 - z^2 - 2z}$ around

(i) $z_0 = 1$

(ii) $1 < |z+1| < 3$

(iii) $|z+1| > 3$. 5

8. (a) Evaluate : 5

$$\int_0^\infty \frac{\cos 5x}{x^2+4}\, dx$$

(b) Evaluate : 5

$$\int_{-\infty}^{\infty} \frac{dx}{(x^3-1)^2}$$