

Registration No. :

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Total number of printed pages – 3

B. Tech
BSCM 1205

Third Semester Regular Examination – 2014

MATHEMATICS – III

BRANCH(S) : AEIE, AERO, AUTO, BIOMED, BIOTECH, CHEM,
CIVIL, CSE, EC, EEE, EIE, ELECTRICAL, ENV, ETC,
FASHION, IEE, IT, MANUTECH, MECH, MINERAL,
MINING, MM, MME, PLASTIC, TEXTILE

QUESTION CODE : H 371

Full Marks – 70

Time – 3 Hours

Answer Question No. 1 which is compulsory and any **five** from the rest.

The figures in the right-hand margin indicate marks.



1. Answer the following questions :

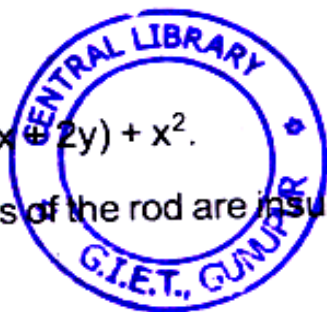
2 × 10

- What is the difference between a general solution and a complete solution ?
- What is Dirichlet problem ?
- Find the vibration of the string of length 1 and $c = 1$ starting with initial velocity zero and initial deflection $f(x) = k(x - x^3)$.
- What do you know about types of Partial differential equations ? State the type of the equation $u_{xx} + 5u_{xy} + 2u_{yy} = 0$.
- Find the fixed points of the mapping $w = \frac{5z + 4}{z + 5}$.

- (f) Find, whether $f(z)$ is continuous at $z = 0$ if $f(z) = \begin{cases} \frac{\operatorname{re}(z^2)}{|z|^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$.

P.T.O.

- (g) Define convergence, absolute convergence, conditional convergence of a series with giving example.
- (h) Check whether the following sequence is bounded, convergent and find its limit point : $z_n = \frac{(1+i)^{2n}}{2^n}$.
- (i) State Morera's theorem.
- (j) Explain different types of singularity giving examples.
2. (a) Solve : $p^2q(x^2 + y^2) = p^2 + q$. 5
 (b) Solve : $(x + pz)^2 + (y + qz)^2 = 1$. 5
3. (a) Solve : $4D^2 - 4DD' + D'^2 = 16 \log(x + 2y) + x^2$. 5
 (b) Solve the heat equation when ends of the rod are insulated using variable separable method. 5
4. (a) Solve $u_{xx} + u_{yy} = 0$ which satisfies the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin(n\pi x/l)$. 5
 (b) Find the vibrations of a rectangular membrane of sides $a = 4$ ft and $b = 2$ ft if the tension is 12.5 lb/ft, the density is 2.5 slugs/ft², the initial velocity is 0 and the initial displacement is $f(x, y) = 0.1(4x - x^2)(2y - y^2)$. 5
5. (a) Are the following functions analytic? 5
 (i) $f(z) = z^2 + \frac{1}{z^2}$
 (ii) $f(z) = xy + ix^2y$
 (b) Find the analytic function whose real part is $e^x \sin(x^2 - y^2)$. 5
6. (a) Evaluate the integral $\int_c (z+1)^2 dz$ where c is the boundary of the rectangle with vertices at the point $2 + 3i, -2 + 3i, -2 - 3i, 2 - 3i$. 5
 (b) Evaluate $\int_c \frac{2z^2 + 5}{(z+1)^3(z^2+3)} dz$ where c is a circle having center at 2 and radius 5. 5



7. (a) Find the Taylor's series expansion of $f(z) = ze^{2z}$ around $z = -1$, find the radius of convergence of the above. 5

(b) Find the Laurent's series expansion of $f(z) = \frac{7z-2}{z^3 - z^2 - 2z}$ around

(i) $z_0 = 1$

(ii) $1 < |z+1| < 3$

(iii) $|z+1| > 3$.

8. (a) Evaluate:

$$\int_0^{\infty} \frac{\cos 5x}{x^2 + 4} dx$$

(b) Evaluate:

$$\int_{-\infty}^{\infty} \frac{dx}{(x^3 - 1)^2}$$



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