

Registration No. :

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Total number of printed pages – 3

B. Tech  
BS 1104

Second Semester Regular Examination – 2014

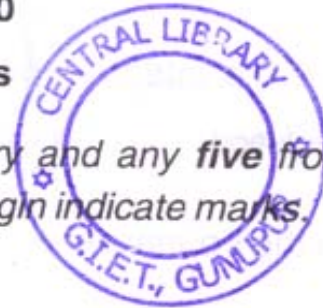
MATHEMATICS - II

BRANCH(S) : ALL

QUESTION CODE : F 459

Full Marks – 70

Time : 3 Hours



Answer Question No. 1 which is compulsory and any **five** from the rest.  
The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10
- (a) Find the Laplace transform of an unit step function.
  - (b) State convolution theorem.
  - (c) Define Fourier integral representation.
  - (d) Write the formula to find Fourier sine integral of any function.
  - (e) Express  $\beta$ -function in terms of gamma function.
  - (f) Prove that  $\operatorname{erf} f + \operatorname{erf} f_c = 1$ .
  - (g) Write the geometrical significance of grad of a function.
  - (h) Write the formula to find the arc length of the curve C.
  - (i) Write the parametric representation of the ellipsoid  $x^2 + y^2 + (1/9)z^2 = 1$ .
  - (j) What is the significance of Gauss divergence theorem.
2. (a) Find the inverse Laplace transform of the following : 5

(i) 
$$\frac{e^{-15s}}{(s-3)(s-5)}$$

(ii) 
$$\frac{s^3 - s^2 - s + 4}{s^4 - 5s^2 + 4}$$

P.T.O.

- (b) Solve the following system of differential equations using Laplace transform : 5

$$y_1' = 2y_1 + 4y_2$$

$$y_2' = y_1 + 2y_2$$

$$y_1(0) = -4, y_2(0) = -4$$

3. (a) Solve the integral equation,  $y(t) = \sin 2t + \int_0^t \sin 2(t-x)y(x) dx$ , using convolution. 5

- (b) Find the Fourier series expansion of  $f(x) = \pi \sin \pi x$ ,  $0 < x < 1$ . 5

4. (a) Find the Fourier transform of  $f(x) = e^{-2x}$  when  $x \geq 0$  and 0 otherwise. 5

- (b) Show that  $\int_0^{\infty} \frac{x^3 \sin xw}{x^4 + 4} dx = \frac{\pi}{2} e^{-x} \cos x$  if  $x \geq 0$ . 5

5. (a) Find a tangent vector and its corresponding unit tangent vector of the curve

$$\mathbf{r}(t) = \cosh t \mathbf{i} + 2\sinh t \mathbf{j} \text{ at the point } P(1/3, 4/3, 0). \quad 5$$

- (b) Find the constants  $a$  and  $b$  so that the surface  $ax^2 + byz = (a+2)x$  will be orthogonal to the surface  $4x^2y + z^3 = 4$  at the point  $(1, -1, 2)$ . 5

6. (a) Prove that  $\text{div}(\mathbf{f} \times \mathbf{g}) = \text{curl} \mathbf{f} \cdot \mathbf{g} - \text{curl} \mathbf{g} \cdot \mathbf{f}$  5

- (b) Find the directional derivative of  $f = 4e^{2x-y+z}$  at the point  $(1, 1, -1)$  in the direction towards the point  $(-3, 5, 6)$ . 5

7. (a) Evaluate the integral  $I = \int 3x^2 dx + 2zy dy + y^2 dz$  from  $A(0, 1, 2)$  to  $B(1, -1, 7)$  by showing that  $F$  has a potential and hence  $I$  is independent of path. 5

(b) Find the moment of inertia of a lamina  $S : x^2 + y^2 = z^2, 0 \leq z \leq h$  of density 1 about the z-axis. 5

8. (a) Using Green's theorem, find the area bounded by the hypocycloid  $x^{2/3} + y^{2/3} = a^{2/3}$  with  $a > 0$ . 5

(b) Verify Stokes theorem for  $\mathbf{f} = y^2 \mathbf{i} + z^2 \mathbf{j} + x^2 \mathbf{k}$  and  $S$  is the upper half of the sphere  $x^2 + y^2 + z^2 = 1$  and  $y \geq 0, z \geq 0$ . 5

