Registration No. : | | | | | | | | | | |

Total number of printed pages – 3

B. Tech
BS 1104

Second Semester Regular Examination – 2014

MATHEMATICS - II

BRANCH(S) : ALL

QUESTION CODE : F 459

Full Marks – 70

Time : 3 Hours

*Answer Question No. 1 which is compulsory and any **five** from the rest.*
The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

   (a) Find the Laplace transform of an unit step function.

   (b) State convolution theorem.

   (c) Define Fourier integral representation.

   (d) Write the formula to find Fourier sine integral of any function.

   (e) Express β-function in terms of gamma function.

   (f) Prove that $er\,f + er\,f_c = 1$.

   (g) Write the geometrical significance of grad of a function.

   (h) Write the formula to find the arc length of the curve C.

   (i) Write the parametric representation of the ellipsoid $x^2 + y^2 + (1/9)z^2 = 1$.

   (j) What is the significance of Gauss divergence theorem.

2. (a) Find the inverse Laplace transform of the following : 5

   (i) $\dfrac{e^{-15s}}{(s-3)(s-5)}$

   (ii) $\dfrac{s^3 - s^2 - s + 4}{s^4 - 5s^2 + 4}$

(b) Solve the following system of differential equations using Laplace transform : 5

$$y_1' = 2y_1 + 4y_2$$

$$y_2' = y_1 + 2y_2$$

$$y_1(0) = -4,\ y_2(0) = -4$$

3. (a) Solve the integral equation, $y(t) = \sin 2t + \int_0^t \sin 2(t-x)\, y(x)\, dx$, using convolution. 5

(b) Find the Fourier series expansion of $f(x) = \pi \sin \pi x,\ 0 < x < 1$. 5

4. (a) Find the Fourier transform of $f(x) = e^{-2x}$ when x □ 0 and 0 otherwise. 5

(b) Show that $\int_0^\infty \frac{x^3 \sin xw}{x^4 + 4} dx = \frac{\pi}{2} e^{-x} \cos x$ if x □ 0. 5

5. (a) Find a tangent vector and its corresponding unit tangent vector of the curve

$\mathbf{r}(t) = \cosh t\ \mathbf{i} + 2\sinh t\ \mathbf{j}$ at the point P(1/3, 4/3, 0). 5

(b) Find the constants a and b so that the surface $ax^2 + byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, −1, 2). 5

6. (a) Prove that $\text{div}\,(\mathbf{f} \times \mathbf{g}) = \text{curl}\,\mathbf{f}.\mathbf{g} - \text{curl}\,\mathbf{g}.\mathbf{f}$ 5

(b) Find the directional derivative of $f = 4e^{2x-y+z}$ at the point (1, 1, −1) in the direction towards the point (−3, 5, 6). 5

7. (a) Evaluate the integral $I = \int 3x^2 dx + 2zydy + y^2 dz$ from A (0, 1, 2) to B (1, −1, 7) by showing that F has a potential and hence I is independent of path. 5

(b) Find the moment of inertia of a lamina S : $x^2+y^2=z^2, 0 \leq z \leq h$ of density 1 about the z-axis. 5

8. (a) Using Green's theorem, find the area bounded by the hypocycloid $x^{2/3}+y^{2/3}=a^{2/3}$ with a □ 0. 5

(b) Verify Stokes theorem for $\mathbf{f}=y^2\ \mathbf{i}+z^2\ \mathbf{j}+x^2\ \mathbf{k}$ and S is the upper half of the sphere $x^2+y^2=z$ and $y \geq 0, z \leq 1$. 5
