GIET UNIVERSITY, GUNUPUR – 765022

RN19MSC024

Roll No: | | | | | | |

Total Number of Pages : 2 AR-18 M.SC

M.Sc 3rd SEMESTER REGULAR EXAMINATIONS, NOV/DEC 2019-20

Subject code: CE-313

Subject: Ordinary Differential Equations-I

Time: 3 Hours Max Marks: 80

The figures in the right hand margin indicate marks.

SECTION A

Q.1 Answer any four of the following: [4 x 4 =16]

a Show that $\phi(t)=\frac{t^3}{3}+\frac{t^2}{2}+2t+1$ is a solution of the differential equation $x''=2t+1,\ x(0)=1,\ x'(0)=2$ 4

b Solve $\left(ut+tx^2\right)dt+\left(x+t^2x\right)dx=0$ by the method of separation variables. 4

c Let f_1 and f_2 be linearly independent on an interval I. Prove that the functions f_1+f_2 and f_1-f_2 are also linearly independent on I. 4

d Solve the system of equations: 4

$$x_1' = 5x_1 - 2x_2,\ x_2' = 2x_1 + x_2$$

e Compute the first three successive approximations for the solution of $x'=e^x,\ \ x(0)=0$ 4

f Verify that $y(t)=\cos t$ is a solution of $y'(t)=y\left(t-\frac{3}{2}\pi\right)$ 4

(OR)

2. Answer all questions from the following [8 x 2 =16]

a Determine the order of the differential equation 2

$$\frac{d^2x}{dt^2}+7\frac{dx}{dt}+\int xdt=t^3$$

b Show that the equation $x'=f(at+bx+c)$ is transformed into the equation $u'=bf(u)+a$ by the substitution $u=at+bx+c$ 2

c Solve $x''+4x'+20x=0;\ \ -\infty<t<\infty$. 2

d Define Wronskian. 2

e Define solution matrix. 2

f State the necessary and sufficient condition for the system $x'=Ax$ to admit a non-zero periodic solution of period ω. 2

g Write the general form of delayed differential equation. 2

h State Contraction Principle. 2

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SECTION-B

3. Answer all Questions: [4 x16 =64]

a(i)	Let $x(t,t_0,x_0)$ and $y(t,t_0,x_0)$ denote the solutions of the equations $g\left(t.x.x'\right) \equiv x' + a(t)x = 0$ and $x' + a(t)x = b(t),\ t \in I$ respectively passing through $\left(t_0,x_0\right)$ and existing on I. Then prove that $y(t,t_0,x_0) = x(t,t_0,x_0) + \int_{t_0}^{t} x\left(t,s,b(s)\right)ds,\ t \in I$	8
(ii)	Solve $\left(xdt - tdx\right) + x\left(t^2 + x^2\right)dx = 0$	8
	OR	
b(i)	State and Prove the necessary and sufficient condition that the equation $M(t,x)dt + N(t,x)dx = 0$ is exact.	8
(ii)	Find the differential equations satisfied by all the parabolas whose axis is the axis of *t*.	8

4.

a(i)	Let $b_1,...b_n$ be real or complex valued functions defined and continuous on an interval I and $\phi_1,\phi_2,...,\phi_n$ are n solutions of the equation $L(x)(t) = x^{(n)}(t) + b_1(t)x^{(n-1)}(t) + ... + b_n(t)x(t) = 0,\ t \in I$. Prove that these n solutions are linearly independent on I if and only if $w(t) \neq 0$ for every $t \in I$.	8
(ii)	Solve $x^{iv} - 2a^2x'' + a^4x = 0$	8
	OR	
b(i)	State and Prove Abel's formula for Wronskian.	10
(ii)	Solve $x'' + x' = 4t^2e^t$	6

5.

a	Determine $\exp(tA)$ for the system $x' = A(t)x$ where $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 3 \\ 0 & 1 & 0 \end{bmatrix}$	16
	OR	
b	Let A(t) be an n x n matrix that is continuous on closed and bounded interval I. Then prove that there exists a solution to the initial value of problem of $x' = Ax$ on I, also prove that the solution is unique.	16

6.

a(i)	Prove that the error $x(t) - x_n(t)$ estimates $\left\|x(t) - x_n(t)\right\| \leq \frac{L(Kh)^{n+1}}{K(n+1)}e^{Kh};\ t \in \left[t_0,\ t_0 + h\right]$	8
(ii)	Let $a < 0$ *and* $0 < b < \|a\|$. Then prove that any solution *x* of $x'(t) = ax(t) + bx(t-r),\ 0 \leq t_0 \leq t < \infty$ tends to zero as $t \to \infty$	8
	OR	
b	State and Prove Picard's theorem	16