GIET UNIVERSITY, GUNUPUR – 765022

RD19MSC002

Roll No: | | | | | | |

Total Number of Pages : 2 AR-19 M.SC

M.Sc 1ST SEMESTER REGULAR EXAMINATIONS, NOV/DEC 2019-20

MTPC101

PARTIAL DIFFERENTIAL EQUATIONS AND ITS APPLICATIONS

Time: 3 Hours Max Marks: 80

The figures in the right hand margin indicate marks.

SECTION A

Q.1 Answer any four of the following:[4 X 4 =16]

a	Find the Fourier transform of $\frac{1}{\lvert x\rvert}$	4 marks
b	Convert the equation into canonical form y^2u_{xx} - $x^2u_{yy} = 0$	4 marks
c	Transform the equation in canonical form $u_{xx} - 4u_{xy} + 4u_{yy} = e^y$	4 marks
d	Determine the general solution of $4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$	4 marks
e	For any y, z belongs to D (L) we have the Lagrange identity y L[z] –z L[y]= $\frac{d}{dx}[p(yz^1 - zy^1)]$	4 marks
f	If the Laplace transform of f(t) is F(s) then the Laplace transform of f(ct) with c>0 is (1/c)F(s/c)	4 marks

OR

2. Answer all questions from the following [2 x 8 =16]

a	Define order, homogeneous and non homogeneous partial differential equation	2 marks
b	Define linear and quasi linear PDE	2 marks
c	Define Jacobian and Canonical forms	2 marks
d	State Cauchy- Kowalewskaya theorem	2 marks
e	Explain about Wave equation	2 marks
f	Define Fourier transforms in PDE and give one use	2 marks
g	Explain about Convolution theorem in PDE	2 marks
h	Define Laplace Transform and conditions of Laplace transforms	2 marks

SECTION-B

3. Answer all Questions: [16 x 4 =64]

a Convert the PDF to canonical form: x^2u_{xx} + 2xy $u_{xy} + y^2u_{yy} = 0$ 16 marks

Find the characteristics equation and reduce it into canonical form

$$u_{xx} \mp sechx^4u_{yy} = 0$$

Or

b Construct the Green's function for two point BVP 16 marks

$y'' + \omega^2 y = f(x), \quad y(a) = y(b) = 0.$

4.

a Determine the solution of the following problem 16 marks

$u_{tt} = c^2 u_{xx}$ 0< x< l, t>0 $u(x, 0) = \sin\frac{\pi x}{l}$ $0 < or = x < or = l$

U(x, 0) =0, u(0,t) =0 u(l, t) =0

Or

b State and prove Uniqueness theorem. 16 marks

5.

a Determine the solution of the initial and boundary value problem 16 marks

$u_{tt} = 9\, u_{xx}$, $0 < x < \infty,\ t > 0$

$u(x, 0) = 0,\ 0 \le x < \infty,$

$u_t(x, 0) = x^3,\ 0 \le x < \infty,$

$u_x(0, t) = 0,\ t \ge 0$

)

Or

b 16 marks

Show that i) $F_C\{e^{-ax}\} = \sqrt{\frac{2}{\pi}}\ \frac{a}{a^2+k^2}$, a>0

ii) $F_s\{e^{-ax}\} = \sqrt{\frac{2}{\pi}}\ \frac{k}{a^2+k^2}$, a>0

$iii) F_s - 1\left\{\frac{1}{k}e^{-sk}\right\} = \sqrt{\frac{2}{\pi}}$ tan-1($\frac{x}{s}$), a>0

6.

a i) Prove that the Fourier transform F is linear 16 marks

ii) Let F [f(x)] be a Fourier transform of f(x) then prove that

F [f(x-c)]=$e^{-ikc}\ F(f(x))$

Or

b Show that the solution of the Dirichlet's problem, if it exists is unique. 16 marks