GIET UNIVERSITY, GUNUPUR – 765022

RD19MSC032

Roll No: | | | | | | |

Total Number of Pages : 2 AR-19 M.SC

M.Sc 1ST SEMESTER REGULAR EXAMINATIONS, NOV/DEC 2019-20

MTPC104

ELEMENTARY COMPLEX ANALYSIS

Time: 3 Hours Max Marks: 80

The figures in the right hand margin indicate marks.

SECTION A

Q.1 Answer any four of the following: [4 X4 =16]

a Find all the values of $\left(\frac{1}{2}+\frac{\sqrt{3}}{3}i\right)^{\frac{3}{4}}$. 4 Marks

b Find the general value of log(-i). 4 Marks

c Show that the function f(z) = $\sqrt{|xy|}$ is not analytic at the origin even though C-R equations are satisfied thereof. 4 Marks

d Show that $w=\frac{i-z}{i+z}$ maps the real axis of z-plane into the circle |w| = 1 and the half plane $y > 0$ into the interior of the unit circle |w| = 1 in the w-plane. 4 Marks

e State and prove Morera's theorem (or) Converse of Cauchy's theorem. 4 Marks

f Show that the series $z(1-z)+z^2(1-z)+z^3(1-z)+\cdots\infty$ converges for |z| < 1. Determine whether it converges absolutely or not. 4 Marks

OR

2. Answer all questions from the following [2 x 8 =16]

a Find the value of $(1+2i)^3$ 2 Marks

b Find the fourth roots of unity. 2 Marks

c Separate the real and imaginary parts of cosh(x+iy). 2 Marks

d State Cauchy – Riemann equations in polar form. 2 Marks

e Find the critical points of the transformation $w=z+\frac{1}{z}$. 2 Marks

f Find the fixed points of the transformation $w=\frac{z}{2z-1}$. 2 Marks

g State Cauchy's fundamental theorem. 2 Marks

h Find the nature and location of the singularities of the function $f(z)=\frac{e^{2z}}{(z-1)^4}$ 2 Marks

SECTION-B

Answer all the following questions :- [16 x4 =64]

3.

a i)Find the cube roots of unity and show that they form an equilateral triangle in the Argand diagram. 16 Marks

ii) Use Demoivre's theorem to solve the equation $x^4-x^3+x^2-x+1=0$

OR

b i)Expand $cos^8\theta$ in a series of cosines of multiples of θ. 16 Marks

ii) Expand $sin^7\theta cos^3\theta$ in a series of sines of multiples of θ.

4.

a Prove that the necessary and sufficient conditions for the derivative of the function $w = f(z) = u(x,y) + iv(x,y)$ to exist for all values of z in a region R, are i) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ are continuous functions of x and y in R — 16 Marks

ii) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.

OR

b Determine the analytic function $f(z) = u + iv$, if $u - v = \frac{cosx+sinx-e^{-y}}{2(cosx-coshy)}$ and $f\left(\frac{\pi}{2}\right) = 0$. — 16 Marks

5.

a Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = 2, i, -2$. Hence find the fixed and critical points of this transformation. — 16 Marks

OR

b Transform $i)$ $w = e^z$ ii) $w = coshz$ — 16 Marks

6.

a State and prove Cauchy's integral formula and hence evaluate $\oint_C \frac{sin\pi z^2+cos\pi z^2}{(z-1)(z-2)}dz$ where C is the circle |z| = 3. — 16 Marks

OR

b (i) State and prove Liouville's theorem. — 16 Marks

(ii) Evaluate $\int_c \frac{dz}{(z+4)(z+1)^3}$, where $C: |z| = 3$.