GIET UNIVERSITY, GUNUPUR – 765022

RD19MSC022

Roll No: | | | | | | |

Total Number of Pages : 2 AR-19 M.Sc

M. Sc 1ST SEMESTER REGULAR EXAMINATIONS, NOV/DEC 2019-20

MTPC103 : ALGEBRA – I

Time: 3 Hours Max Marks: 80

The figures in the right hand margin indicate marks.

SECTION A

Answer any FOUR of the following: [4 x 4 = 16]

1. a. Show that $N(a)$ is a subgroup of G

b. If $O(G) = p^2$, where p is prime number, then prove that G is abelian.

c. If p is prime number of the form $4n$ +1, then prove that $p = a^2 + b^2$ for some integers a and b.

d. Let R be an Euclidean ring. Suppose that for $a, b, c \in R$, $a|bc$ but $[a, b]$ =1, then prove that $a|c$.

e. Show that $L[S]$ is a subspace of V.

f. If $f(x) \in F[x]$ is irreducible, then prove that

(i) If the characteristic of F is 0, $f(x)$ has no multiple roots.

(ii) If the characteristic of F is $p \neq 0$, $f(x)$ has a multiple root only if it is of the form$f(x) = g(x^p)$.

(OR)

Answer all questions from the following. [2 x 8 = 16]

2.a. Let $\sigma = (1 \ 5 \ 3)(1 \ 2)$, $\tau = (1 \ 6 \ 7 \ 9)$, then compute $\sigma\tau\sigma^{-1}$.

b. Define conjugate of an element in G.

c. State second part of Sylow's theorem.

d. Define Euclidean ring.

e. Define linearly dependent vectors and linearly independent vectors.

f. Prove that $x^2 + 1$ is irreducible over R.

g. Define (i) algebraic extension (ii) algebraic number.

h. What is the degree of $x^5 - 1$ over Q?

SECTION – B [4 x 16 = 64]

3. a. State and prove Cauchy's theorem.

(OR)

b. (i). State and prove Cayley's Theorem.

(ii) Show that conjugacy is an equivalence relation on G.

4.a. If $p^{\alpha} | O(G)$ and $p^{\alpha+1} \nmid O(G)$, then G has a subgroup of order p^{α}.

(OR)

b. Prove that $J[i]$ is a Euclidean ring.

5. a.(i). State and prove Eisenstein Criterion.

(ii) Prove that for any prime p, $x^{p-1} + x^{p-2} + \cdots + x + 1$ is irreducible over rational.

(OR)

b. If V is finite dimensional and if W is a subspace of V, then prove that W is finite dimensional, $\dim W \le \dim V$ and $dim\left(\frac{V}{W}\right) = \dim V - \dim W$

6.a. If L is a finite extension of K and if K is a finite extension of F, then L is a finite extension of F. Moreover, $[L : F] = [L : K][K : F]$

(OR)

b. (i). For any $f(x), g(x) \in F[x]$ and any $\propto \in F$, then prove that (1) $[f(x) + g(x)]' = f'(x) + g'(x)$ (2) $[\propto f(x)]' = \propto f'(x)$ and (3) $[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$.

(ii) If F is a field of characteristic $p \neq 0$, then the polynomial $x^{p^n} - x \in F[x]$, for $n \geq 1$ has distinct roots.