Registration No :

Total Number of Pages : 03

B.Tech
PCS3I001

3rd Semester Back Examination 2019-20
DISCRETE STRUCTURES
BRANCH : CSE
Max Marks : 100
Time : 3 Hours
Q.CODE : HB820

Answer Question No.1 (Part-1) which is compulsory, any EIGHT from Part-II and any TWO from Part-III.
The figures in the right hand margin indicate marks.

Part-I

Q1 Only Short Answer Type Questions (Answer All-10) **(2 x 10)**

a) Show that $\neg (p \vee \neg q)$ and $(q \wedge \neg p)$ are equivalent by using rules of logical equivalences.

b) Prove that the sum of two even integers is always even.

c) Consider the function $f(n) = 2\lfloor n/2 \rfloor$ from Z to Z. Is this function one-to-one? Is this function onto? Justify your answers.

d) Write a recursive algorithm for computing (3^2^n), where n is a non-negative integer and ^ is an exponentiation operator.

e) Derive a generating function for the sequence 2, 3, 4, 5, ……………..

f) Find the solution of the recurrence relation $a_n = 3\, a_{(n-1)}$, with $a_0 = 2$.

g) Suppose that $f(n)$ satisfies the divide-and-conquer relation $f(n) = 2f(n/3) + 5$ and $f(1) = 7$. Write value of $f(81)$.

h) How many non-isomorphic simple graphs are there with three vertices? Draw examples of each of these.

i) Consider the graphs K_5, $K_{2,3}$ and W_5.
Which of these graphs have an E circuit?
Which have an Euler path?

j) Find all the proper subgroups of S_3, the symmetric group of degree 3.

Part-II

Q2 Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve) **(6 x 8)**

a) Using predicate logic to prove that: $(\forall x)\,[P(x) \rightarrow Q(x)] \wedge (\forall x)P(x) \rightarrow (\forall x)Q(x)$, clearly show each step in derivation process.

b) Prove that in the Fibonacci sequence: $F(n + 4) = 3F(n + 2) - F(n)$ for all $n \geq 1$.

c) Define reflexive, symmetric, and transitive closure of a relation.
Find the reflexive, symmetric, and transitive closure of the relation:
R={(*a*, *a*), (*b*, *b*), (*c*, *c*), (*a*, *c*), (*a*, *d*), (*b*, *d*), (*c*, *a*), (*d*, *a*)} on the set *S* = {*a*, *b*, *c*, *d*}.

d) Illustrate the concept of poset. Consider the relation "*x* divides *y*" on {1, 2, 3, 6, 12, 186}.
- i) Write the ordered pairs (*x*, *y*) of this relation.
- ii) Write all the predecessors of 6.
- iii) Write all the immediate predecessors of 6.
- iv) Draw the Hasse diagram for the above relation.

e) Define generating function of the numeric function. Determine the generating function of the numeric function a_r, Where $a_{r} = 2^r$ if r is even and $a_{r} = -2^r$ if r is odd.

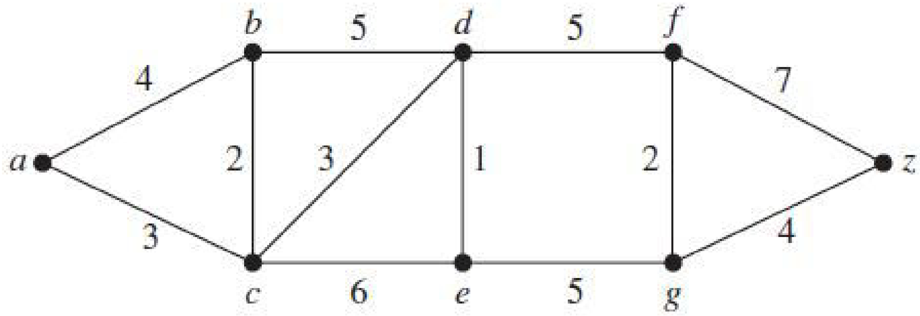
f) Use mathematical induction to show that
$1 + 2 + 2^2 + \ldots\ldots + 2^n = 2^{n+1} - 1$,
For all nonnegative integers *n*.

g) Using generating functions to derive an expression for finding the number of ways to select *r* objects of *n* different kinds if at least one object of each kind is must selected.

h) Let *R* be the relation on the set of real numbers such that *xRy* if and only if *x* and *y* are real numbers that differ by less than 1, that is, $|x - y| < 1$. Show that *R* is not an equivalence relation. Also prove by taking a suitable counter example.

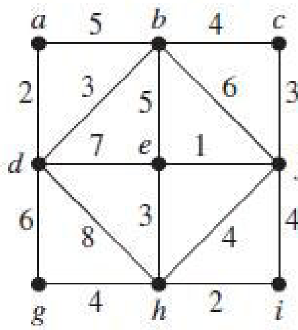
i) Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ for all sets *A*, *B*, and *C* by using set identities and by using membership table.

j) Using Dijkstra's algorithm, Find the length of a shortest path between *a* and *z* in the given weighted graph below:



Clearly specify each intermediate steps in derivation of final tree.

k) Using Prim's algorithm and Kruskal's algorithm to find a minimum spanning tree for the given weighted graph and draw the final tree in each case with clearly mentioning each intermediate step:



l) If $Z_5 = \{0, 1, 2, 3, 4\}$ and Addition modulo 5 is defined as $+_5$, on Z_5 by rule $x +_5 y = r$, where r is the remainder when x + y is divided by 5, means that, $x +_5 y = (x + y) \bmod 5$ and Multiplication modulo 5 is defined as $x \bullet_5 y = (x \bullet y) \bmod 5$. Then prove that, $[Z_5, +_5]$ is a commutative group, and $[Z_5, \bullet_5]$ is a commutative monoid, where Z is set of integers.

Part-III

Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 Describe Principle of Inclusion and Exclusion. **(16)**

Prove that if *A*, *B*, and *C* are sets, then

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$.

Find the number of primes less than 100 using the principle of inclusion–exclusion.

Q4 Describe the terms: scaling factor, accumulated sum, forward difference, backward difference, and convolution with respect to discrete numeric function. **(16)**

Let a, b and c be three discrete numeric functions such that: a = 3r-2, b = (2/r) + 7, c = r ln r, where ln is log base 2 and answer the questions given below by justifying each.

(a) Does a dominate b asymptotically?
Does a dominate c asymptotically?
Does b dominate a asymptotically?
Does b dominate c asymptotically?
Does c dominate a asymptotically?
Does c dominate b asymptotically?
(b) Does a + b dominate a asymptotically?
Does a + b dominate c asymptotically?
(c) Does ab dominate a asymptotically?
Does a dominate ab asymptotically?
Does ab dominate c asymptotically?
Does c dominate ab asymptotically?
(d) Does Aa dominate b asymptotically?
(e) Does the accumulated sum of a dominate c asymptotically?

Q5 Discuss the terms: literal, minterm, maxterm, SOP, POS with respect to Boolean Algebra. Show that the Boolean operator NOR, i.e., { ↓ } is functionally complete. **(16)**
Use K-map method to minimize the sum-of-product expression:

$$xyz + xyz' + xy'z + xy'z' + x'yz + x'y'z + x'y'z'$$

Q6 **Write notes on ANY TWO :** **(8×2)**

a) Pigeonhole principle and its applications
b) Integral domains and Fields
c) Traveling sales person's problem