Registration No : | | | | | | | | | | |

Total Number of Pages : 02 **B.Tech**

PME3D001

3rd Semester Back Examination 2019-20

APPLIED MATHEMATICS

BRANCH : MECH

Max Marks : 100

Time : 3 Hours

Q.CODE : HB948

Answer Question No.1 (Part-1) which is compulsory, any EIGHT from Part-II and any TWO from Part-III.

The figures in the right hand margin indicate marks.

Part- I

Q1 **Only Short Answer Type Questions (Answer All-10)** **(2 x 10)**

a) What are Parabolic and hyperbolic equations? Give one example in each case

b) Find the singular points of $f(z) = \frac{z^2+1}{z^2-z}$

c) Define Cauchy integral formula

d) Solve the partial differential equation $u_{xy} + u_{yy} = 0$

e) Check whether the function $\sin z$ is analytic or not

f) Determine the probability that at least one head appears in the throw of three fair coins.

g) A book has two misprints per page (on average), what is the probability that a page open at random will have no misprints on it?

h) Define critical region.

i) Express Laplace Equation in Cartesian co-ordinates.

j) Find the mean and variance of Normal Distribution.

Part- II

Q2 **Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve)** **(6 x 8)**

a) Derive Necessary and sufficient condition of Analytic Function

b) Evaluate the integral: $\int_{-\infty}^{\infty} \frac{sinx}{(x^2+4x+5)}\,dx$

c) Integrate the function $f(z) = \frac{z^3+sinz}{(z-i)^2}$, where C is the boundary of the square with vertices ± 2 and $\pm\ 2i$..

d) Solve: $x(r + 2xs + x^2t) = p + 2x^3$ by Monge's method

e) Find the temperature $u(x,y)$ in bar of silver (length 10 cm with constant cross section of area 1 cm/square, density10.6gm/cm square, thermal conductivity 1.04 cal/cm, specific heat 0.056 cal/gm) that is perfectly insulated laterally, whose end are kept at temperature 0℃ is $f(x) = x(10 - x)$.

f) In a sample of 9 observations, the sum of squared deviation of items from the mean was 98.5. In other sample of 10 observations, the value was found by 102.7. Test whether the difference is significant at 5% level of significance?

g) Find a harmonic conjugate of given harmonic function u(x,y)=y^3-3x^2y and analytic function f(z)

h) Solve: $(D^2 + DD' + D' - 1)z = \sin(x + 2y) + x^2 + y^2$

here $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$.

i) Check the nature of singularity of the following function:

(i) $f(z) = \frac{\cos z + \sin z}{z^2}$ (ii) $f(z) = e^{\frac{1}{z}}$.

j) Transform the equation $u_{xx} - 4u_{xy} + 3u_{yy} = 0$ to normal form using suitable transform and solve it.

k) A function defined as follows :

$$f(x) = \begin{cases} 4x(9 - x^2) & ,0 \le x \le 3 \\ 0 & otherwise \end{cases}$$

(i) Show that it is a density function.
(ii) Find the expectation and variance.
(ii) Find the distribution function.

l) Assume that on average one telephone number out of ten called between 10A.M to 11 A.M. on week is busy. If five telephone numbers are randomly selected find the probability that
(i) Not more than two
(ii) At least two of them will be busy.

Part-III
Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 **a)** Find all Taylor and Laurent Series of $f(z) = \frac{-2z + 3}{z^2 - 3z + 2}$ with center zero. **(8)**

b) Evaluate counter clockwise $\oint_c \frac{30z^2 - 23z + 5}{(2z-1)^2.(3z-1)} dz \quad C : |z| = 1$ **(8)**

Q4 **a)** Show that Normal distribution is the limiting case of binomial distribution under the following conditions. **(8)**
(i) n, the number of trials is indefinitely large.i.e n→∞
(ii) Neither p (probability of success) nor q(probability of failure) is very small.

b) Suppose an item is inspected at the end of each day to see whether it is still functioning property. Let p be the probability of failure during each day. Find the maximum number of days of inspection for first failure. **(8)**

Q5 **a)** Evaluate the integral: $\int_0^{2\pi} \frac{1 + \sin\theta \, d\theta}{3 + \cos\theta}$. **(8)**

b) State and prove D-Alembert's solutions of wave equation. **(8)**

Q6 By using Least square method for the following data, find out : **(16)**
a) Regression line of y on x
b) Regression line of x on y
c) Fit a parabola $y = ax^2 + bx + c$

x	:	1	2	3	4	5	6
y	:	6	4	3	5	4	2