Registration No. : | | | | | | | | | | |

Total number of printed pages – 2

MCA

MCC 103

Special Examination – 2012

DISCRETE MATHEMATICS

Full Marks – 70

Time : 3 Hours

Answer Question No. ***1*** *which is compulsory and any* ***five*** *from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following : 2×10

(i) Find the matrix representation of the relation R = {(a, a), (a, b), (b, c)} defined on {a, b, c}.

(ii) Define an equivalence relation. Do you think the relation R = {(a,a), (b,c), (c,a), (b,a)} defined on the set A ={ a , b, c} is an equivalence relation ?

(iii) What do you mean by reflexive closure of a relation ? What is the reflexive closure of an equivalence relation R defined on a set A ?

(iv) What are the characteristics of Hasse diagram of a partial order relation ?

(v) Define maximal and minimal elements of a lattice.

(vi) Find the inverse of each element of the poset (P(S), $\subseteq$), where S is the set {a, b, c}.

(vii) Define semigroup and monoid. Give one example of each.

(viii) Define the degree of a vertex of a graph and length of a path in a graph.

(ix) Differentiate between a Hamiltonian path and Euler path.

(x) What is the structure of a proof by contradiction ?

2. (a) Prove by mathematical induction that $4^n - 1$ is divisible by 3 for all integers n > 0. 5

(b) Let R be a relation on a set A. Explain how to use the diagraph of R to create the diagraph of the symmetric closure of R. 5

3. (a) If R is a relation defined on $A = \{a_1, a_2, a_3, \ldots, a_n\}$, then show that $M_{R^2} = M_R \otimes M_R$ 5

(b) Let R be a relation on a set A. Then show that R^∞ is the transitive closure of R. 5

4. (a) Show that if n is a positive integer and $p^2 \mid n$, where p is a prime number, then D_n is not a Boolean algebra. 5

(b) Show that in a Boolean algebra for a, b, c if $a \leq b$ then $a \vee (b \wedge c) = b \wedge (a \vee c)$. 5

5. (a) Define a tree. Construct a tree of the following algebraic structure : 4
$((2 + x) - (2 \times x)) - (x - 2)$

(b) Let R be a symmetric relation on a set A. Then show that the following statements are equivalent : 6

(i) R is an undirected tree

(ii) R is connected and acyclic

6. (a) Let G be a group with identity element e. Show that if $a^2 = e$ for all a in G, then G is abelian. 5

(b) Let G be a group. Show that the function $f : G \to G$ defined by $f(a) = a^2$ is a homomorphism if and only if G is abelian. 5

7. (a) Show that if a graph G has more than two vertices of odd degree then there can be no Euler path in G. 5

(b) What do you mean by spanning tree of a graph ? Write one algorithm to find the minimum spanning tree of a graph and explain each step. 5

8. Write short notes of the following : 2.5×4

(a) Tree traversal

(b) Group code

(c) Methods of proof

(d) Chromatic Number