B.Tech
BSCM 1211

Fourth Semester Special Examinations, 2012
DISCRETE MATHEMATICS
Full Marks – 70
Time : 3 Hours

Answer question No. 1 which is compulsory and any five from the rest.
The figures in the right-hand margin indicate marks

1.Answer the following questions precisely 2x10

(a) Show that $(p \to q) \to r$ and $p \to (q \to r)$ are not equivalent .

(b) Find the truth value of $\exists$ x P(x) ,where P(x) is the statement '$x^2 > 8$ ' and the universe of discourse consists of the positive integers not exceeding 3.

(c) Prove that for every positive integer n , n^3 + n is even.

(d) Find the generating function for the sequence (1,1,1,1,1,1).

(e) Find the distance between the words x = 1100010 and y = 1010001.

(f) Prove that each element a in a group G has only one inverse in G.

(g) Investigate the lexicographic ordering , constructed from the relation $\leq$ on Z of (1,3,5,7) $<$ (1,3,7,9) .

(h) Is the poset A = {2,3,6,12,24,36,72} under the relation of divisibility a lattice ? If so what are the maximal and minimal elements ?

(i) State true or false : In a graph , an Euler path can also be a Hamiltonian path in some cases, If true, then give an example of such case , else disprove this statement.

(j) What is the principle of duality on a lattice.

2.(a) Find the solution of the recurrence relation $a_n = 4\,a_{n-1} - 3\,a_{n-2} + 2^n + n + 3$ with the initial condition a_0 = 1 and a_1 = 4. 5

(b) Prove by induction that any positive integer n greater than or equal to 2 is either a prime or a product of primes. 5

3. (a) Write the systematic procedure of Warshall's algorithm. Let A = {1,2,3,4} and R= {(1,4),(2,1),(2,3),(3,1),(3,4),(4,3)} be a relation defined on A. Use Warshall's algorithm to find the transitive closure of R. 5

(b) Prove that a finite poset has

(i) at most one greatest element

and (ii) at most one least element. 5

4. (a) Let S = {1,2,3,4} and A = S x S. Define the following relation R on A : (a,b) R (a', b') if and only if a + b = $a' + b'$.Then show that R is an equivalence relation and determine the partitions for A. 5

(b) A computer company receives 350 applications from computer graduates for a job, planning for a new web servers. Suppose that 220 of these people majored in computer science, 147 majored in business and 51 majored both in computer science and business. How many of these applicants majored neither in computer science nor in business ? 5

5. (a) What is the minimum number of vertices in a 3-regular bipartite planar graph ? 5

(b) Show that the maximum number of edges in a simple undirected graph with n vertices is $\frac{n(n-1)}{2}$. 5

6. (a) Prove that every connected undirected graph has a spanning tree. 5

(b) If T is a spanning tree of a connected graph G and e is an edge not in T , then prove that, inclusion of e to T makes a cycle. 5

7.(a) If (G , $*$) is a group with identity e and if a $*$ a = e for all a $\in$ G , then show that G is abelian. 5

(b) Let (S , $*$) and (T , $*'$)be monoids with identities e and e' respectively. Let f : S $\rightarrow$ T be an isomorphism .Then prove that f(e) = e' . 5

8. (a) Prove that every field is an integral domain. 5

(b) Prove that, a complemented distributive lattice establishes De Morgan's Law. 5