Registration No. :

Total number of printed pages – 3

Special Examination – 2012

MATHEMATICS – III

Full Marks – 70

Time : 3 Hours

Answer Question No. **1** which is compulsory and any **five** from the rest.

The figures in the right-hand margin indicate marks.

1.   Answer the following questions :   2×10

(a)   Define Linear partial differential equation.

(b)   Solve the partial differential equation $q + s + t = 0$

(c)   Write the two dimensional wave equation explaining initial conditions and boundary conitions.

(d)   Whether a function is differentiable at a point without being analytic there. Explain giving an example.

(e)   State the Cauchy Integral theorem for multiple connected domain.

(f)   Define fixed point. Find the fixed points of Mobius transformation.

(g)   Find the Maclaurin series expansion of $f(z) = Ln(1 + z)$.

(h)   Define Uniform convergence of a series.

(i)   Find the radius of convergence and region of convergence of the series

$$\sum_0^\infty \frac{n!}{2^n} z^{2n}.$$

(j)   Define isolated singularity, pole and essential singularity of a complex function.

P.T.O.

2. (a) Solve : $\dfrac{y^2 z}{x} p + xzq = y^2$ 5

(b) Solve : $2x^2 q^2 + 3yq + xp - 2z = 0$. 5

3. (a) Solve : $(D_x^2 D_y + D_y^2 - 2)z = e^{2y} \sin x + e^x \cos 2y$. 5

(b) Solve : $(x^2 D_x^2 - xyD_x D_y + 2y^2 D_y^2)z = \log\dfrac{x}{y}$. 5

4. (a) The imaginary part of an analytic function is $x^2 - y^2 - y$. Find the corresponding analytic function. 5

(b) Find a Mobius transformation which maps unit disk onto the unit disk. 5

5. (a) Evaluate the integral $\displaystyle\int \cos^2 z\,dz$ along $|z| = \pi$ in the right half plane. 5

(b) Evaluate the integral $\displaystyle\int \dfrac{\ln(z+4)}{(z-3)(z-2)^2}\,dz$ where C is the boundary of a square with vertices $\pm 2.5, \pm 2.5\,i$. 5

6. (a) Check the convergence of the following series : 5

(i) $\displaystyle\sum_{n=1}^{\infty} n\sqrt{\dfrac{1}{n}}$

(ii) $\displaystyle\sum_{n=2}^{\infty} \dfrac{1}{\ln(n+1)}$

(b) Find the Taylor series expansion of $\sin 2z^2$ around $z = 0$. 5

7. (a) Determine the type of sibgularity of the following function : 5

(i) $\cosh\left[(z^2 + 2)^{-1}\right]$

(ii) $(z - \pi i)^{-2} \sinh z$

(b) Find the Laurent series expansion of $f(z) = \dfrac{z^3}{(z+3i)^4}$ around $z = -3i$.   5

8. (a) Evaluate : $\displaystyle\int_{-\infty}^{\infty} \dfrac{\sin 3x}{x^4 + 1}\,dx$ 5

(b) Evaluate : $\displaystyle\int_{0}^{2\pi} \dfrac{\sin\theta}{17 - 8\sin\theta}\,d\theta$ 5

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