Registration No. : | | | | | | | | | | |

Total number of printed pages – 2

B. Tech

BSCM 2102(O)

Special Examination – 2012

MATHEMATICS – II

Full Marks – 70

Time : 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

   (a) Check whether the vectors (2,5,0), (5,3,8), (8,6,3) in R are linear independent or not.

   (b) Define eigen value and eigen vector of the matrix.

   (c) Define Hermitian, Skew-Hermitian , Orthogonal, Unitary matrices.

   (d) Find the smallest positive period of the function $f(x) = \sin 2\pi x$ and $\cos 5nx$

   (e) Prove that $\beta(m,n) = \beta(n,m)$.

   (f) Find the projection of a = (5, 3, –1) over b = (2, 2, 1).

   (g) Prove that $\nabla \times \bar{r} = 0$

   (h) Compare Binomial, Poission's and Hypergeometric distribution.

   (i) What is the surface area of the surface S whose equation is F (x, y, z) = 0 ?

   (j) States Green's theorem.

2. (a) Using Gauss elimination method solve the following system of equation : 5

   $2x - y - 3z = 9$, $x + 3y - z = -4$, $3x + y + 4z = 10$.

   (b) Find a matrix A such that $X^TAX = (x_1 - x_2 + 4x_3)^2 - 4(x_2 - x_4)^2$. 5

3. (a) Find the Fourier series expansion of $f(x) = \pi \sin \pi x$, $0 < x \leq 1$ and 0 otherwise. 5

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(b) Find the Fourier sine transform of $f(x) = \sin 2x,\ 0 < x < \pi$ and 0 otherwise. 5

4. (a) Show that : $\int_0^\infty \frac{w^3 \sin \pi w}{w^2 + 4}\, dw = \pi e^{-x} \cos x$. 5

(b) Find the unit normal vector at (1,1,2) to the surface $x^2 + y^2 + 5z^2 = 20$. 5

5. (a) Prove that : $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$. 5

(b) Find the area of the ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 5

6. (a) Prove that : $\text{div}\,(\vec{u} \times \vec{v}) = \vec{v}.\text{curl}\,\vec{u} - \vec{u}.\,\text{curl}\,\vec{v}$. 5

(b) Find directional derivative of $f = xy^2 - 3xyz$ at (1,2,2) in the direction of normal to the surface $x^2 + y^2 - z^2 = 1$ at (1,1,1). 5

7. (a) If S is any closed surface enclosing a volume V and $\mathbf{F} = x\mathbf{i} + 2y\,\mathbf{j} + 3z\mathbf{k}$, find $\int \mathbf{F}.\mathbf{n}\, ds$. 5

(b) Find the integral $\int f.dr$ where $f = (2z, x, -y)$, $r = (\cos t, \sin t, 2t)$ from (1,0,0) to $(1, 0, 4\pi)$. 5

8. (a) Using Gauss divergence theorem, evaluate the integral of $\oiint_S F.n\,dA$ of $F = [x^3, y^3, z^3]$ and S is the sphere $x^2 + y^2 + z^2 = 9$. 5

(b) Verify Green's theorem in the plane for $\int_C \{(2x^2 - y^3)\, dx - xydy)\}$ where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. 5
