Registration No. : | | | | | | | | | | |

Total number of printed pages – 2

B. Tech
BS 1104

Special Examination – 2012

MATHEMATICS – II

Full Marks – 70

Time : 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

(a) What is the condition for existence of Laplace transform ?

(b) Find the inverse Laplace transform of $f(t) = \ln\left(\frac{s+2}{s+3}\right)$.

(c) Find the Laplace transform of unit step function.

(d) Find the smallest positive period of the function $f(x) = \sin 3\pi x$

(e) Prove that $\beta(m,n) = \beta(n,m)$.

(f) Find the projection of $a = (5, 3, -1)$ over $b = (2, 2, 1)$.

(g) Prove that $\nabla \times \bar{r} = 0$

(h) Find the parametric representation of the Ellipsoid $x^2 + y^2 + \frac{1}{4}z^2 = 1$

(i) What is the surface area of the surface S whose equation is $F(x, y, z) = 0$.

(j) State Stokes theorem.

2. (a) Solve the differential equation $y^{11} + 3y^1 + 2y = t$, $y(0) = 0$ and $y^1(0) = 1$ using Laplace equation. 5

(b) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. 5

3. (a) Find the Fourier series expansion of $f(x) = x$, when $0 < x < \pi$ 5

$= \pi - x$ when $\pi < x < 2\pi$.

P.T.O.

(b) Find the Fourier sine transform of $f(x) = \sin 2x$, $\quad 0 < x < \pi$ 5

$\qquad = 0 \qquad$ otherwise.

4. (a) Show that : $\displaystyle\int_0^{\infty} \frac{w^3 \sin \pi w}{w^2 + 4}\, dw = \pi e^{-x} \cos x.$ 5

(b) Show that : $\Gamma(1/2) = \sqrt{\pi}$ 5

5. (a) Prove that : $\nabla^2 f(r) = f''(r) + \dfrac{2}{r} f'(r)$ 5

(b) Find the area of the ellipse : $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1.$ 5

6. (a) Prove that : $\text{div}\,(\vec{u} \times \vec{v}) = \vec{v}.\text{curl}\,\vec{u} - \vec{u}.\,\text{curl}\,\vec{v}$ 5

(b) Find directional derivative of $f = xy^2 - 3xyz$ at $(1,2,2)$ in the direction of normal to the surface $x^2 + y^2 - z^2 = 1$ at $(1,1,1)$. 5

7. (a) If S is any closed surface enclosing a volume V and $\mathbf{F} = x\mathbf{i} + 2y\,\mathbf{j} + 3z\mathbf{k}$. Find $\int \mathbf{F.n}\, ds$. 5

(b) Find the integral $\int f.dr$ where $f = (2z, x, -y)$, $r = (\cos t, \sin t, 2t)$ from $(1,0,0)$ to $(1, 0, 4\pi)$. 5

8. (a) Using Gauss divergence theorem, evaluate the integral of $\int\int_S F.\,ndA$ of $F = [x^3, y^3, z^3]$ and S is the sphere $x^2 + y^2 + z^2 = 9$. 5

(b) Verify Green's theorem in the plane for $\int_C \{(2x^2 - y^3)\,dx - xydy)\}$ where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. 5
