

Registration No. :

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Total number of printed pages – 2

B. Tech
BS 1101

Special Examination – 2012

MATHEMATICS – I

Full Marks – 70

Time : 3 Hours

Answer Question No. 1 which is compulsory and any **five** from the rest.

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10
- (a) Define Linear differential equation.
 - (b) Define Wronskian and find the wronskian of $y_1 = \cos x$, $y_2 = \sec x$.
 - (c) Do you think the differential equation $(x^3 + xy^2) dx + (ax^2y + bxy^2) dy = 0$ exact ? If not, then what shall be the integrating factor to make it exact ?
 - (d) Define the asymptote of a curve.
 - (e) What are Bessel's function and Bessel's equation ?
 - (f) Show that $J_1^1(x) = J_0(x) - \frac{1}{x} J_1(x)$.
 - (g) Define symmetric matrix, skew-symmetric matrix, Hermitian matrix, orthogonal matrix.
 - (h) What kind of conic section is represented by the quadratic form $6x_1^2 + 16x_1x_2 - 6x_2^2 = 0$
 - (i) Prove that eigen value of a symmetric matrix is real.
 - (j) Prove that if λ is an eigen value of a matrix A then $\frac{1}{\lambda}$ is an eigen value of A^{-1} .
2. (a) Solve : $\frac{dy}{dx} = x^3y^2 + xy$ 5
- (b) Solve $(3x^2y^4 + 2xy) dx + (2x^3y^3 - x^2) dy = 0$. 5

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3. (a) Find the current in the simple circuit with $c = \infty$ and $E(t) = E_0 \sin \omega t$. 5
- (b) Solve : $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 4y = e^x \cos x$. 5
4. (a) Solve the differential equation $(1 - x^2)y'' - 2xy' + 2y = 0$, given that $y_1 = x$ is a solution. 5
- (b) Using variation of parameter to solve the differential equation $\frac{d^2y}{dx^2} + 9y = \sec 3x$. 5
5. (a) Find the radius of curvature of the curve $r^2 = a^2(1 - \cos^2 \theta)$. 5
- (b) Find the asymptotes of the curve $y^2 = \frac{x^2(a+x)}{a-x}$. 5
6. (a) Find the power series solution of $y'' + 8xy' - 4y = 0$. 5
- (b) Show that : $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ 5
7. (a) If A and B are two square matrices of the same order and A is symmetric then prove that $B^T A B$ is also symmetric. 5
- (b) Solve the system of equation : 5
- $$\begin{aligned} 2x - 3y &= 1 \\ 2x - y - z &= 2 \\ 3x + y - 2z &= 1 \end{aligned}$$
8. (a) Find the eigen value and its corresponding eigen vector of the following matrix : 5
- $$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$
- (b) Diagonalize the following matrix after finding the basis vectors : 5
- $$A = \begin{bmatrix} -43 & 77 \\ 13 & 93 \end{bmatrix}$$

