Registration No. :

Total number of printed pages – 2

Sixth Semester Examination – 2013

ADVANCED NUMERICAL METHODS / NUMERICAL METHODS

BRANCH : CHEMICAL ENGINEERING

QUESTION CODE : A 266

Full Marks – 70

Time : 3 Hours

*Answer Question No. 1 which is compulsory and any five from the rest.*
The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

   (a) Define spline function .

   (b) What is linear and quadratic interpolation ?

   (c) Write the central difference formula to find $f'(x_i)$, $f''(x_i)$, $f'''(x_i)$, $f''''(x_i)$.

   (d) What is QR method ?

   (e) Find $f''(3)$ of the following data using central difference formula :

x	1	2	3	4	5
y	0.5	2	5	8	17

   (f) Define Accelerating convergence.

   (g) What is a Predictor–corrector method ?

   (h) Obtain the finite difference scheme for solving the differential equation
   $$2 \frac{d^2y}{dx^2} + y = 5 \, .$$

   (i) Write the Crank–Nicolson's formula to solve the heat equation.

   (j) Define elliptic, parabolic and hyperbolic type of partial differential equations.
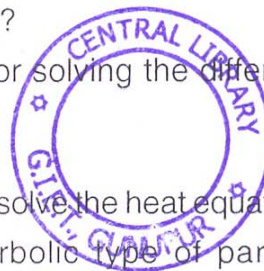
P.T.O.

2. (a) Find the piecewise quadratic polynomial for the following data points : 5
   $(1, 3)$, $(0, -2)$, $(1, -4)$, $(2, 6)$.

   (b) Using cubic-spline interpolation, compute $y(1.2)$ of the given data. 5

x	1	2	3	4
y	1.5	2.2	3.1	4.3

3. (a) Estimate the value of $f'(\pi/2)$ for $f(x) = \cos x/x$, using Richardson's extrapolation method taking central difference formula as base method.
   5

   (b) Using Romberg integration, evaluate $\int_{2}^{2} \frac{x}{1+x} dx$ taking $h = 0.2$. 5

4. (a) Find the dominant eigen value of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & -2 \end{bmatrix}$. 5

   (b) Find the smallest eigen value of the matrix $\begin{bmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix}$. 5

5. (a) Find the best-fit trigonometric polynomial of degree $m = 1$ of the following data : 5

x	0	$\pi/2$	π	$3\pi/2$
y	1	1	0	0

   (b) Using FFT, find the interpolation function for the data $z = \{0, 1, 2, 3\}$. 5

6. (a) Using Adams-Moulton 3rd order, find $y(0.6)$ of the initial value problem $dy/dx = y^2 - x\,y$, $y(0) = 0.4$.
   5

   (b) Using Adam-Bashforth of 4th order Predictor-Corrector method, solve the initial value problem $dy/dx = y - 2x/y$, $y(0) = 1$ in the interval $[0, 1]$. 5

7. Using implicit method, solve the heat equation $u_t - u_{xx} = 0$, for $0 < x < 1, t > 0$. The initial conditions are $u(x, 0) = x^2$, for $0 < x < 1$ with boundary conditions are $u(0, t) = 0$, $u(1, t) = 1$, for $t > 0$ for 3 time step. 10

8. Using explicit method, solve the wave equation $u_{tt} - u_{xx} = 0$, for $0 < x < 1, t > 0$. The initial conditions are $u(x, 0) = 2x$, for $0 < x < 1$, with boundary conditions are $u(0, t) = 0$, $u(1, t) = 0$. 10