Registration No. : | | | | | | | | | | |

Total number of printed pages – 3

B. Tech

FESM 6302 (New)

Sixth Semester (Back) Examination – 2013

NUMERICAL METHODS

BRANCH : CHEM

QUESTION CODE : B296

Full Marks – 70

Time : 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*
The figures in the right-hand margin indicate marks.
Assume suitable notations and any missing data wherever necessary.

1. Answer the following questions : 2×10

   (a) What are numerical methods ? Write down its application.

   (b) If $Y(X_{i-1}) = Y_{i-1}$ and $Y(X_i) = Y_i$, write down the piecewise linear interpolation formula for Y(X) valid in $X_{i-1} \le X \le X_i$.

   (c) Write the forward difference formula to find $f'(X_i)$, $f''(X_i)$, $f'''(X_i)$ and $f''''(X_i)$.

   (d) What is Eigen value problem and write down its application.

   (e) Find out Eigen values and eigen vector of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

   (f) Find a QR factorization of $\begin{bmatrix} 3 & 7 \\ 4 & 4 \end{bmatrix}$.

   (g) What is the need of numerical solution for differential equation ?

   (h) What do you mean by partial differential equation ?

   (i) What are the methods you use to solve one dimensional wave equation ?

   (j) "Multistep methods are not self starting." Justify.

2. (a) Calculate the density of 26 % solution of phosphoric acid in water using piece wise quadratic interpolation. The following data are quoted from Perry handbook. 5

Y, Density	1.0764	1.1134	1.2160	1.3350
X, Percentage H_3PO_4	14	20	35	50

P.T.O.

(b) Find the interpolating polynomial for the following data using piecewise cubic hermite interpolation. 5

X	0	1	2
Y	1	3	35
Y'	1	6	81

3. (a) The upward force of air resistance on a falling object is proportional to the square root of the velocity. For this the velocity can be computed as :

$$V(t) = \frac{ds}{dt} = \sqrt{\frac{gm}{C_D}} \tanh\left(t\sqrt{\frac{gC_D}{m}} \right),$$

where, C_D is a second order drag coefficient = 0.25 kg/min, g = 9.81 m/s^2, m = 68.1 kg. Determine how far the object falls in 5 seconds by taking $h_1 = 1$ and $h_2 = 0.5$ by using Simpson's 1/3rd rule and then by Romberg's method. 5

(b) Estimate the first derivative of $f(x) = -0.1x^4 - 0.15x^3 - 0.25x + 1.2$ at x = 0.5 using finite differences and a step size of h = 0.25 and compute the error on the basis of the true value of −0.9125. 5

4. Find the QR factorization of $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$. 10

5. (a) Find out numerically smallest Eigen value of the matrix by inverse power method. $A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$. 5

(b) Find the least square trigonometric approximation with m = 2 for five data points. 5

T	0	$2\pi/5$	$4\pi/5$	$6\pi/5$	$8\pi/5$
X	1	3	2	0	−1

6. (a) Using Milne's predictor corrector method find Y(4.4). Given that $5xy' + y^2 - 2 = 0$, y(4) = 1, y(4.1) = 1.0049, y(4.2) = 1.0097 and y(4.3) = 1.0143. 5

(b) A mass balance for a chemical in a completely mixed reactor can be written as :

$$V\frac{dc}{dt} = F - Qc - KVc^2$$

where, V = Volume = 12 m^3,

C = concentration (g/m^3),

F = Feed rate = 175 g/min,

Q = flow rate = 1 m^3/min, and

K = second order reaction rate = 0.15 m^3/g/min.

If C(0) = 0 and h = 0.5, then solve the ODE using 3rd order Adam's Bashforth method until the concentration reaches a stable level. 5

7. (a) Using Crank-Nicolson method, solve $U_t = U_{xx}$, subject to u(x, 0) = 0, u(0, t) = 0, and u(1, t) = t, by taking

(i) h = 0.5 and k = 1/8 and

(ii) h = 1/4 and k = 1/8. 5

(b) Derive explicit method to find the solution of a one dimensional wave equation (Hyperbolic equation). 5

8. Write short notes on any **two** : 5×2

(a) Richardson's extrapolation

(b) Discrete Fourier transforms

(c) Finite element method

(d) Spline interpolation.