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Total number of printed pages – 3

B. Tech
FESM 6302

Sixth Semester Regular Examination – 2014

NUMERICAL METHODS

BRANCH : CHEM

QUESTION CODE : F 318

Full Marks – 70

Time : 3 Hours

Answer Question No. 1 which is compulsory and any **five** from the rest.

The figures in the right-hand margin indicate marks.

Assume suitable notations and any missing data wherever necessary.

1. Answer the following questions : 2 × 10
- (a) What are numerical methods ? Write down its application.
- (b) If $Y(X_{i-1}) = Y_{i-1}$ and $Y(X_i) = Y_i$, write down the piecewise linear interpolation formula for $Y(X)$ valid in $X_{i-1} \leq X \leq X_i$.
- (c) Write the forward difference formula to find $f'(X_i)$, $f''(X_i)$, $f'''(X_i)$ and $f^{(4)}(X_i)$.
- (d) What is Eigen value problem and write down its application.
- (e) Find out Eigen values and eigen vector of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.
- (f) Find a QR factorization of $\begin{bmatrix} 3 & 7 \\ 4 & 4 \end{bmatrix}$.
- (g) What is the need of numerical solution for differential equation ?
- (h) What do you mean by partial differential equation ?
- (i) What are the methods you use to solve one dimensional wave equation ?
- (j) "Multistep methods are not self starting." Justify.
2. (a) Calculate the density of 26% solution of phosphoric acid in water using piece wise quadratic interpolation. The following data are quoted from Perry handbook. 5

Y, Density	1.0764	1.1134	1.2160	1.3350
X, Percentage H_3PO_4	14	20	35	50

P.T.O.

- (b) Find the interpolating polynomial for the following data using piecewise cubic hermite interpolation. 5

X	0	1	2
Y	1	3	35
Y'	1	6	81

3. (a) The upward force of air resistance on a falling object is proportional to the square root of the velocity. For this the velocity can be computed as :

$$V(t) = \frac{ds}{dt} = \sqrt{\frac{gm}{C_D}} \tanh \left(t \sqrt{\frac{gC_D}{m}} \right),$$

where, C_D is a second order drag coefficient = 0.25 kg/min, $g = 9.81 \text{ m/s}^2$, $m = 68.1 \text{ kg}$. Determine how far the object falls in 5 seconds by taking $h_1 = 1$ and $h_2 = 0.5$ by using Simpson's 1/3rd rule and then by Romberg's method. 5

- (b) Estimate the first derivative of $f(x) = -0.1x^4 - 0.15x^3 - 0.25x + 1.2$ at $x = 0.5$ using finite differences and a step size of $h = 0.25$ and compute the error on the basis of the true value of -0.9125 . 5

4. Find the QR factorization of $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$. 10

5. (a) Find out numerically smallest Eigen value of the matrix by inverse power

method $A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$. 5

- (b) Find the least square trigonometric approximation with $m = 2$ for five data points. 5

T	0	$2\pi/5$	$4\pi/5$	$6\pi/5$	$8\pi/5$
X	1	3	2	0	-1

6. (a) Using Milne's predictor corrector method find $Y(4.4)$. Given that $5xy' + y^2 - 2 = 0$, $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$ and $y(4.3) = 1.0143$. 5

- (b) A mass balance for a chemical in a completely mixed reactor can be written as :

$$V \frac{dc}{dt} = F - Qc - Kvc^2$$

where, $V = \text{Volume} = 12 \text{ m}^3$,

$C = \text{concentration (g/m}^3\text{)},$

$F = \text{Feed rate} = 175 \text{ g/min},$

$Q = \text{flow rate} = 1 \text{ m}^3\text{/min}, \text{ and}$

$K = \text{second order reaction rate} = 0.15 \text{ m}^3\text{/g/min}.$

If $C(0) = 0$ and $h = 0.5$, then solve the ODE using 3rd order Adam's Bashforth method until the concentration reaches a stable level. 5

7. (a) Using Crank-Nicolson method, solve $U_t = U_{xx}$, subject to $u(x, 0) = 0$, $u(0, t) = 0$ and $u(1, t) = t$, by taking

(i) $h = 0.5$ and $k = 1/8$ and

(ii) $h = 1/4$ and $k = 1/8$. 5

- (b) Derive explicit method to find the solution of a one dimensional wave equation (Hyperbolic equation). 5

8. Write short notes on any **two** : 5×2

- (a) Richardson's extrapolation
- (b) Discrete Fourier transforms
- (c) Finite element method
- (d) Spline interpolation.

