Registration No. : | | | | | | | | | | |

Total number of printed pages – 3

B. Tech

FESM 6302

Fifth Semester Examination – 2013

ADVANCED NUMERICAL METHODS

BRANCH : MM, MECH, MME, CIVIL

QUESTION CODE : C- 310

Full Marks – 70

Time : 3 Hours

*Answer Question No. **1** which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

(a) Define spline function.

(b) Define piecewise interpolation and explain its importance.

(c) Find the fourth derivative at x = 4 of the following data :

x	0	2	4	6	8
y	2	5	8	14	17

(d) Explain Richardson's Extrapolation.

(e) Explain shifted power method.

(f) What is Fast Fourier transform ?

(g) Write the formula for Milne-Simphson predictor- corrector method.

(h) Write the stability condition for explicit method and implicit method in wave equation.

(i) When a method is said to be stable ?

(j) What is the importance of Crank-Nicolson formula to solve the heat equation ?

P.T.O.

2. (a) Find a Hermite interpolating polynomial for the following data points : 5

x	0.4	0.5	0.7	0.8
$f(x)$	−0.9162	−0.6931	−0.3566	−0.2231
$f'(x)$	2.50	2.00	1.43	1.25

(b) Find a natural cubic spline function for the data values : 5

x	0	1	2	3	4
y	1	1	0.56	0.36	0.26

3. (a) Find dy/dx, d^2y/dx^2 at $x = 1.0$ and for $x = 2.0$ of the following data : 5

x	1	1.2	1.4	1.6	1.8	2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891

(b) Using Romberg integration, evaluate $I = \int_0^1 \frac{\log(x+1)}{\sqrt{x(1-x)}}\, dx$. 5

4. (a) Write the different steps of QR factorization to find the eigenvalue of a matrix. 5

(b) Find eigenvalue of the following matrix using Rayleigh Quotient method : 5

$$A = \begin{bmatrix} 2 & 2 & 1 \\ -5 & 9 & -3 \\ -4 & 4 & 1 \end{bmatrix}$$

5. (a) Find the Fourier approximating polynomial of the following data : 5

x	0	$\pi/2$	π	$3\pi/2$	2π
y	0	1/4	1/2	3/4	1

(b) Using FFT, find the interpolation function for the data $z = \{0, 1, 2, 3\}$. 5

6. (a) For $\frac{dy}{dx} = \frac{y}{x^3 - 1}$, $y(1) = 0$, find $y(1.4)$ using Adams-Moulton method. 5

(b) Given that $dy/dx = 4e^{0.8x} - 0.5y$, $y(0) = 2$. Find $y(2)$ using ABM 3rd order predictor-corrector method. 5

7. Using Crank-Nicolson method ,solve the following heat equation $u_t - u_{xx} = 0$, for $0 < x < 1$, $t \leq 0.75$ with the initial condition $u(x, 0) = \sin \pi x$ and boundary conditions are $u(0, t) = 0$, $u(0, t) = 1$. 10

8. Using explicit method, solve the wave equation $u_{tt} - u_{xx} = 0$, for $0 < x < 1$, $t > 0$. The initial conditions are $u(x, 0) = x^2$, $u_t(x, 0) = 0$, for $0 < x < 1$ with boundary conditions $u(0, t) = 0$, $u(0, t) = 1$, for $t > 0$. (Take $h = 0.2$ and $k = 0.2$) 10