Registration No : | | | | | | | | | | |

Total Number of Pages : 02 **B.Tech**
PAT2A001

2nd Semester Back Examination 2018-19
APPLIED MATHEMATICS - II
BRANCH : AEIE, AERO, AUTO, BIOTECH, CHEM, CIVIL, CSE, ECE, EEE, EIE, ELECTRICAL, ENV, ETC, IEE, IT, MANUTECH, MECH, METTA, MINERAL, MINING, MME, PE, PLASTIC, PT, TEXTILE
Max Marks : 100
Time : 3 Hours
Q.CODE : F127

Answer Question No.1 (Part-1) which is compulsory, any EIGHT from Part-II and any TWO from Part-III.
The figures in the right hand margin indicate marks.

Part- I

Q1 **Only Short Answer Type Questions (Answer All-10)** **(2 x 10)**

a) Using definition of Laplace transformation, determine $L\{e^{at}\}$, when $s > a$.
b) Find $L\{e^{at}t^n\}$.
c) The function $f(x) = cos(x)$ is even function or odd function or neither even nor odd. Justify your answer.
d) Write the Fourier series of $f(x)$ in the interval $(-\pi, \pi)$.
e) Write the relation between beta and gamma function.
f) If $\vec{r} = x^2\hat{\imath} + y^2\hat{\jmath} + z\hat{k}$, then find $div(\vec{r})$.
g) Check whether the vector $\vec{v} = yz\hat{\imath} + zx\hat{\jmath} + xy\hat{k}$ is irrotational or not.
s Find directional derivative of $f = sin\, yz + xyz$ at $P:(-1,1,3)$ in the direction $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$.
i) State Stokes's Theorem.
j) Determine values of a_0 and a_n in the Fourier expansion of $f(x) = sinx, -\pi < x < \pi$.

Part- II

Q2 **Only Focused-Short Answer Type Questions- (Answer Any Eight out of Twelve)** **(6 x 8)**

a) Develop Laplace transform of the piecewise continuous function

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 2 - t, & 1 \le t < 2 \\ 0, & t \ge 2 \end{cases}$$

b) Calculate $L\{\int_0^t \int_0^t \int_0^t cosau\, du\, du\, du\}$.
c) Solve the integral equation $y(t) = t + \int_0^t y(u)\, sin(t-u)\, du$.
d) Calculate Fourier Cosine transform and Fourier Sine transformation of

$$f(x) = \begin{cases} 1, & 0 \le x < 1 \\ 0, & x \ge 1 \end{cases}$$

e) Develop Fourier series for $f(x) = |x|, -2 < x < 2, p = 4$.
f) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.
g) Calculate $\int_0^{\frac{\pi}{2}} \frac{\sqrt[3]{sin^8(x)}}{\sqrt{cos(x)}}\, dx$.
h) For any scalar function f(x,y,z) and vector function $\vec{v} = [v_1, v_2, v_3]$, Prove that
i. $Curl(grad\, f) = 0$
ii. $Div(curl\, \vec{v}) = 0$

i) Calculate length of the curve $\vec{r}(t) = a\cos t\,\hat{\imath} + a\sin t\,\hat{\jmath} + 4t\,\hat{k}$ from $(a,0,0)$ to $(a,0,8\pi)$.

j) Use Green's theorem to evaluate the line integral $\oint_c -y^3dx + x^3dy$ where c is the circle $x^2 + y^2 = 1$.

k) Develop Half range Fourier Cosine series of the function $f(x) = x^2$, $0 < x < 2$.

l) Calculate the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{\imath} + (2xz - y)\hat{\jmath} + z\,\hat{k}$ along a straight line from $A(0,0,0)$ to $B(2,1,3)$.

Part-III

Only Long Answer Type Questions (Answer Any Two out of Four)

Q3 Verify Green's theorem in plane for $\oint_c (x^2 - 2xy)dx + (x^2y + 3)dy$ where c is the boundary of the region defined by $y^2 = 8x$ and $x = 2$ **(16)**

Q4 Use convolution theorem to evaluate $L^{-1}\{\frac{1}{s^2(s^2+1)}\}$. **(16)**

Q5 Solve using Laplace transform technique **(16)**

$$y'' + 4y = 9t, \qquad y(0) = 0,\ y'(0) = 7$$

Q6 **a)** Using Fourier integral representation, show that **(10)**

$$\int_0^\infty \frac{\cos x\omega + \omega \sin x\omega}{1 + \omega^2} d\omega = \begin{cases} 0, & if\ x < 0 \\ \frac{\pi}{2}, & if\ x = 0 \\ \pi e^{-x}, & if\ x > 0 \end{cases}$$

b) Evaluate Fourier series of $f(x) = x^2, -\pi < x < \pi$. **(6)**