Registration No : | | | | | | | | | | |

Total Number of Pages : 02 **B.Tech**

BS1104

2nd Semester Back Examination 2018-19

MATHEMATICS - II

BRANCH : CIVIL, CSE, ECE, EEE, EIE, ELECTRICAL, IT, MECH, PLASTIC

Time : 3 Hours

Max Marks : 70

Q.CODE : F044

Answer Question No.1 which is compulsory and any FIVE from the rest.

The figures in the right hand margin indicate marks.

Q1 **Answer the following questions :** **(2 x 10)**

a) Determine the Laplace Transform of $f(t) = (t+1)^2 e^t$

b) Derive the parametric representation of the straight line through the point $A(4,2,0)$ in the direction of the vector $b = \hat{\imath} + \hat{\jmath}$

c) What is the divergence of the vector $v = e^x(cosy\,\hat{\imath} + siny\hat{\jmath})$

d) If $f(x,y) = x^2 \cos y$ then what is the value of $\nabla^2 f$ at $(0,0)$.

e) Find $\nabla^2 f$ *where* $f = e^{2x} cos2y$.

f) State Dirac's delta function.

g) State the functions which are even, odd or neither even or odd out of the following functions

$$x + x^2, \quad \ln x \ , \quad x\sin x\ , \quad |x|$$

h) Find curl of of the vector $v = yz\hat{\imath} + 3zx\hat{\jmath} + z\hat{k}$ at the point *(0 ,2, 5)*

i) Derive the unit normal vector to the surface $x^2 + y^2 + z^2 = 1$

j) Using Green's theorem find area of an ellipse.

Q2 **a)** Using Laplace transformation, solve the equation **(5)**

$$y'' + y = r(t), \qquad r(t) = t \ if \ 1 < t < 2 \ and \ 0 \ otherwise.$$

$$y(0) = 0, \qquad Y'(0) = 0$$

b) Show that the form under the integral sign is exact in the plane and evaluate the integral **(5)**

$$\int_{(0,-1,1)}^{(2,4,0)} e^{x-y+z^2}(dx - dy + 2zdz)$$

Q3 **a)** Find the directional derivative of the function $f = \ln(x^2 + y^2)$ at the point $P(4,5)$ in the direction of the vector $a = \hat{\imath} - \hat{\jmath}$ **(5)**

b) Using Convolution, calculate the value of $L^{-1}\left[\frac{1}{s^2(s^2+1)}\right]$ **(5)**

Q4 **a)** Using Gamma function evaluate $\int_0^\infty x^6 e^{-3x} dx$. **(5)**

b) Find the Fourier Transformation of $f(x) = \begin{cases} xe^{-x}, & x > 0 \\ 0, & x < 0 \end{cases}$ **(5)**

Q5 **a)** Find the Fourier cosine integral of $f(x) = e^{-kx} \quad (x > 0, k > 0)$ **(5)**

b) Find the Fourier series of the function $f(x) = 2x, \ (-1 < x < 1)$ with period 2. **(5)**

Q6 Using the Fourier series of $f(x) = \frac{x^2}{2}, \quad (-\pi < x < \pi)$ Prove that **(10)**

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots \ldots\ldots\ldots = \frac{\pi^2}{6}$$

Q7 Define stokes theorem and evaluate the line integral $\int_C F.r'(s)ds$ where **(10)**

$$F = [4z, \quad -2x, \quad 2x], \; C: \text{the ellipse} x^2 + y^2 = 1. \quad z = y + 1$$

Q8 **Write short answer on any TWO :** **(5 x 2)**

a) What is Jacobian and why it is used. Also write its polar form.

b) Find the value of $\Gamma(5/2)$.

c) Find the volume of the region beneath $z = 4x^2 + 9y^2$ and above the rectangle with vertices $(0,0)$, $(3,0)$, $(3,2)$, $(0,2)$