MA/M.Sc.—Math-IVS(404)

2019

Time : 3 hours

Full Marks : 80

Answer from **both** the Sections as per direction

The figures in the right-hand margin indicate marks

Candidates are required to answer in their own words as far as practicable

(OPTIMIZATION TECHNIQUES - II)

SECTION – A

1. Answer any *four* of the following : 4×4

    (*a*) What is quadratic programming ?

    (*b*) What is the difference between linear programming and non-linear programming ?

    (*c*) What is a nomial and what is a signomial ?

    (*d*) What is geometric programming ?

    (*e*) What do you mean by cutting plane method ?

(*Turn Over*)

(*f*) What do you mean by complementarity problem ?

Or

2. Answer *all* the questions : 2×8

(*a*) Define basic variable.

(*b*) What is feasible solution to a LPP ?

(*c*) What is objective function ?

(*d*) What is degenerate solution ?

(*e*) When a matrix is positive semidefinite ?

(*f*) What is artificial variable ?

(*g*) When a matrix is symmetric ?

(*h*) What is gradient of a function ?

SECTION – B

Answer all questions : 16×4

3. (*a*) Minimize $x_1^2 - 2x_1x_2 + 2x_2^2 - 2x_1 - 5x_2$

subject to $2x_1 + 3x_2 \le 20,$

$3x_1 - 5x_2 \le 4$

$x_1 \ge 0, \; x_2 \ge 0$

Solve the above problem by Wolfe's method.

Or

(*b*) Maximize $x_1 + x_2 - \frac{1}{2}x_1^2 + x_1x_2 - x_2^2$

subject to $x_1 + x_2 \le 3$

$2x_1 + 3x_2 \ge 6$

$x_1 \ge 0, x_2 \ge 0$

Solve the above problem by Beale's method.

4. (*a*) Let G be an $n \times n$ positive semidefinite matrix. Prove that, for any $\in > 0$, the matrix $G + \in I$ is positive definite.

Or

(*b*) Consider the linear complementarity problem with a slack variable x_0 in R^m :

$$\begin{pmatrix} W \\ W_0 \end{pmatrix} = \begin{pmatrix} M & A \\ 0 & B \end{pmatrix} \begin{pmatrix} x \\ x_0 \end{pmatrix} + \begin{pmatrix} q \\ 0 \end{pmatrix} \ge 0,$$

$$x^T W + x_0^T W_0 = 0, \; x \ge 0, x_0 \ge 0,$$

where A is an $n \times m$ matrix and B is an $m \times m$ matrix, suppose that $x^T Bx \neq 0$ whenever $0 \leq x \neq 0$. Prove that x solves the problem

$$Mx + q \geq 0, x \geq 0, x^T(Mx + q) = 0$$

if and only if $\begin{pmatrix} x \\ x_0 \end{pmatrix}$ with $x_0 = 0$ solves the foregoing slack linear complementarity problem.

5. (*a*) Solve the following problem with Frank-Wolfe method :

$$\text{Minimize } -2x_1 - 4x_2 + x_1^2 + x_2^2$$
$$\text{subject to } \begin{aligned} 2x_1 + 3x_2 &\leq 6 \\ x_1 + 4x_2 &\leq 5 \\ x_i &\geq 0 \; (i = 1, 2) \end{aligned}$$

Or

(*b*) Solve the following problem by the reduced gradient method :

$$\text{Maximize } x_1^2 - 2x_1 - x_2$$
$$\text{subject to } \begin{aligned} x_1 + 5x_2 &\leq 12 \\ x_1 + x_2 &\geq 2 \\ x_1 \geq 0, x_2 &\geq 0 \end{aligned}$$

(Continued)

6. (*a*) Minimize $g_0(x) = 2\pi\, x_1^2 + 2\pi\, x_1 x_2 + 2\pi\, x_1^{-1}\, x_2$

$$\text{subject to } \begin{aligned} g_1(x) \equiv 16x_1^{-2}x_2^{-1} &\leq 1, \\ x_1 > 0, \; x_2 &> 0. \end{aligned}$$

Or

(*b*) Use the dynamic programming technique. Show that the range of a projectile fired in vacuum is $2\dot{x}_0\dot{y}_0/g$, where $\dot{x}_0$ and $\dot{y}_0$ are the initial velocity components and g is the acceleration due to gravity.
