

2019

Time : 3 hours

Full Marks : 80

Answer from both the Sections as directed

*The figures in the right-hand margin indicate marks**Candidates are required to answer in their own words
as far as practicable***(GRAPH THEORY)****SECTION – A**

1. Answer any *four* of the following : 4 × 4
- (a) List all the self-complementary graphs with 4 or 5 vertices.
 - (b) Show that an edge e of a graph G is a bridge if and only if e is not part of any cycle in G .
 - (c) Give an example of an Euler graph which is randomly traceable from all of its vertices.

- Or*
- (b) Let G be a non-empty graph with at least two vertices. Then prove that G is bipartite if and only if it has no odd cycles.

(2)

- (d) Prove that in any tree with two or more vertices there are at least two pendant vertices.
- (e) Show that K_5 , the complete graph on five vertices is non-planar.
- (f) Explain fusion of two vertices in a graph.

Or

2. Answer all questions : 2 × 8

- (a) Define edge-disjoint subgraph with an example.
- (b) Find the number of edges in a complete graph of S_0 vertices K_{S_0} .
- (c) Define eccentricity of a vertex in a graph.
- (d) Define a k-connected graph.
- (e) Define a Separable graph with an example.
- (f) Draw the graph having the following matrix as adjacency matrix :

(3)

$$\begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 2 \\ 0 & 2 & 1 & 2 \\ 0 & 2 & 2 & 1 \end{vmatrix}$$

- (g) Define a binary tree.
- (h) Prove that the number of odd vertices in a graph is always even.

SECTION – B

Answer all questions : 16 × 4

3. (a) State and prove Euler's formula for planar graphs.

Or

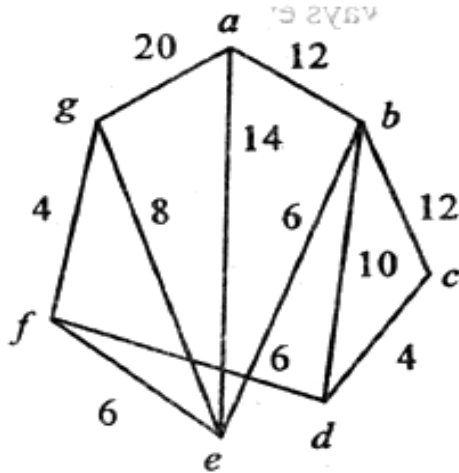
- (b) (i) Let u and v be two vertices of the 2-connected graph G . Then there is a cycle passing through both u and v prove it.
- (ii) Prove that a connected graph is Euler if and only if the degree of every vertex is even.

(4)

4. (a) Prove that the number of labelled trees with n vertices ($n \geq 2$) is n^{n-2} .

Or

- (b) Using Dijkstra's algorithm. Find the shortest path from vertex a to each of the other vertices in the following graph :



5. (a) (i) Show that the numbers of internal vertices in a binary tree is one less than the number of pendent vertices.

(5)

- (ii) Find an expression for the minimum and maximum possible height of a binary tree with n vertices.

Or

- (b) Show that the structure of the saturated hydrocarbon is a tree. How many isomers does pentane C_5H_{12} have ?

6. (a) Define a complete bipartite graph. Give an example of a complete bipartite graph in which.

- (i) Hamiltonian circuit exists but Euler circuit does not exist.
(ii) Hamiltonian circuit does not exist but Euler circuit exists.
(iii) Both Hamiltonian circuit and Euler circuit exist.
(iv) Both Hamiltonian circuit and Euler circuit do not exist.