# MA/M.Sc.—Math-IIS(203)

**2019**

*Time* : 3 hours

*Full Marks* : 80

Answer from **both** the Sections as per direction

*The figures in the right-hand margin indicate marks*

*Candidates are required to answer in their own words as far as practicable*

## ( ALGEBRA - II )

### SECTION – A

1. Answer any *four* of the following questions : 4 × 4

   (*a*) If $W_1$ and $W_2$ are subspaces of a finite dimensional vector space $V_1$ and $\mathscr{A}(W)$ is the annihilator of $W$, then describe $\mathscr{A}(W_1 + W_2)$ in terms of $\mathscr{A}(W_1)$ and $\mathscr{A}(W_2)$.

   (*b*) Define an orthonormal set of vectors. If $\{V_i\}$ is an orthonormal set then show that the vectors in $\{V_i\}$ are linearly independent

( Turn Over )

(*c*) If $\lambda \in F$ is a characteristic root of $T \in A(V)$, then show that $\lambda$ is a root of the minimal poly nomial of $T$.

(*d*) In $T$ is unitary and if $\lambda$ is a characteristic root of $T$, then show that $|\lambda| = 1$.

(*e*) If $\{v_1, v_2, ..., v_n\}$ is an orthonormal basis of $V$ and if the matrix of $T \in A(V)$ in the basis is $(\alpha_{ij})$ then the matrix $T^*$ in this basis is $(\beta_{ij})$ where $(\beta_{ij})\ \overline{\alpha}_{ij}$.

(*f*) If $F$ is of characteristic 0 and if $S$ and T, $A_F(V)$, are such the $ST - TS$ commutes with $S$, then show that $ST - TS$ is nilpotent

*Or*

2. Answer *all* questions from the following : 2 × 8

(*a*) Define annihilator from a subspace of a vector space.

(*b*) Define norm of a vector in an inner product space.

(*c*) What do you mean by the Galois group of a polynomial ?





(*d*) When the group is said to be solvable ?

(*e*) Compute the matrix product

$$\begin{pmatrix} 1 & 6 \\ -6 & 1 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 2 & 3 \end{pmatrix}.$$

(*f*) Give an example of $A$ such that $AA' \neq A'A$.

(*g*) If $A$ is invertiable then show that $\det(ABA^{-1}) = \det B$, for all $B$.

(*h*) Define a normal linear transformation.

## SECTION – B

Answer all questions : 16 × 4

3. (*a*) Prove that the element $a \in K$ is algebraic over $F$ if and only if $F(a)$ is a finite extension of $F$.

*Or*

(*b*) State and prove Schwarz inequality in an inner product space.

4. (*a*) If $A$ is an algebra with unit element over $F$,

then prove that $A$ is isomorphic to a subalgebra of $A(V)$ for some vector space $V$ over $F$.

*Or*

(*b*) Let $G = S_n$ where $n \geq 5$ then show that $G^{(k)}$ for $k = 1, 2, ....$ contains every 3-cycle of $S_n$. Hence prove that $S_n$ ; $n \geq 5$ is not solvable.

5. (*a*) Prove that the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$$

is nilpotent, Find its invariants and Jordan form.

*Or*

(*b*) If $V$ is finite dimensional vector space over $F$, then for $S, T \in A(V)$ prove that

(*i*) $r(ST) \leq r(T)$





(*ii*) $r(TS) \leq r(T)$

(*iii*) $r(ST) = r(TS) = r(T)$ for $S$ regular in $A(V)$.

6. (*a*) Prove that the linear transformation $T$ on $V$ is unitary if and only if it takes an orthonormal basis of $V$ into an orthonormal basis of $V$.

*Or*

(*b*) (*i*) If $N$ is normal then prove that $N^* = p(N)$ for some polynomial $p(x)$.

(*ii*) If $N$ is normal and if $AN = 0$ then prove that $AN^* = 0$.