# MA/M.Sc.—Math-IVS
# (CE-414)

## 2019

*Time* : 3 hours

*Full Marks* : 80

Answer from **both** the Sections as directed

*The figures in the right-hand margin indicate marks*

*Candidates are required to answer in their own words as far as practicable*

## (ORDINARY DIFFERENTIAL EQUATIONS - II)

### SECTION – A

**1.** Answer any *four* of the following : $4 \times 4$

(*a*) Investigate the stationary point $x = 0, y = 0$ of the system

$$\dot{x} = 2x + y - 5y^2$$
$$\dot{y} = 3x + y + x^3$$

for the stability in the first approximation.

(*b*) Prove that

$$x J_n'(x) = -n J_n(x) + x J_{n-1}(x).$$

(*c*) Find generating function for Legendre polynomial.

(*d*) Describe the phase portrait of the following system :

$$\frac{dx}{dt} = -x, \frac{dy}{dt} = -y.$$

(*e*) What is Lyapunov stability ?

(*f*) Construct the green function for the boundary value problem

$$y'' = -f(x), y(0) = y(1) = 0.$$

*Or*

**2.** Answer *all* questions : $2 \times 8$

(*a*) State Strum's comparison theorem.

(*b*) What do you mean by oscillations of differential equations.

(*c*) Define eigen functions and eigen value of a problem.

(*d*) Define asymptotically stability.

(*e*) What is trajectory stability ?





(*f*) Define adjoint operator.

(*g*) State the Strum-Liouville boundary value problem

(*h*) Define Green's function for a boundary value problem.

SECTION – B

Answer **all** questions : $16 \times 4$

**3.** (*a*) Solve the following equation in series

$$(1-x^2)y'' - 2xy' + 2y = 0$$

*Or*

(*b*) Write Legendre's equation and solve it.

**4.** (*a*) For the eigen-value problem given below, obtain the set of orthogonal eigen functions in the interval $(0, 2c)$

$$X'' + \lambda X = 0, \; X(0) = X(2C), X'(0) = X'(2C).$$

*Or*

(*b*) Consider the string probelm

$$\frac{d^2x}{dt^2} = F(t)$$

$x(0) = x(1) = 0$, then construct the Green's function.

5. (*a*) Write Strum's separation theorem and prove it.

*Or*

(*b*) Write Green's formula and prove it.

6. (*a*) Investigate the trivial solution of the system below for stability :

$$\frac{dx}{dt} = x^5 + y^3, \frac{dy}{dt} = x^3 - y^5.$$

*Or*

(*b*) Determine the nature and stability properties





of the critical point (0, 0) for the following system.

$$\frac{dx}{dt} = 2x, \frac{dy}{dt} = 3y.$$