MA/M.Sc.—Math-IIS (201)

2019

Time : 3 hours

Full Marks : 80

Answer from **both** the Section as per direction

The figures in the right-hand margin indicate marks

Candidates are required to answer in their own words as far as practicable

Symbols used have their usual meaning

(ABSTRACT MEASURE)

SECTION—A

1. Answer any *four* of the following questions : 4 × 4

   (*a*) Define outer measure. Show that if E_1 and E_2 are measurable, then

   $$m(E_1 \cup E_2) + m(E_1 \cap E_2) = mE_1 + mE_2.$$

(*b*) Let $<E_i>$ be a sequence of measurable sets, then

$$m(\bigcup E_i) \le \Sigma\, mE_i.$$

If the sets E_n are pairwise disjoint, then

$$m(\bigcup E_i) = \Sigma\, mE_i.$$

(*c*) Define integrable over the measurable set E. Let f be a non-negative function which is integrable over set E. Then given $\varepsilon > 0$ there is a $\delta > 0$ such that for every set $A \subset E$ with $mA < \delta$ we have

$$\int_n f < \varepsilon.$$

(*d*) Let f be of bounded variation on $[a, b]$. Show that

$$\int_a^b |f'| \le T_a^b(f).$$

(*e*) Prove that

$$\|f + g\|_1 \le \|f\|_1 + \|g\|_1.$$

Or

2. Answer *all* questions from the following : 2×8

(*a*) Show that if E is a measurable set, then each translate $E + y$ of E is also measurable.

(*b*) State Lusin's theorem.

(*c*) Let f be a non-negative measurable function. Show that $\int f = 0$ implies $f = 0$ a.e.

(*d*) State monotone convergence theorem.

(*e*) Define Dini derivatives.

(*f*) Define absolutely continuous.

(*g*) Define Banach space.

(*h*) State Minkowski inequality.

SECTION—B

Answer all questions : 16×4

3. (*a*) State and prove that Egoroff's theorem.

Or

(*b*) Show that the interval (a, ∞) is measurable.

4. (*a*) State and prove that Bounded convergence theorem.

Or

(*b*) A bounded function f on $[a, b]$ is Riemann integrable if and only if the set of points at which f is discontinuous has measure zero justify.

5. (*a*) Let f be an increasing real-valued function on the interval $[a, b]$. Then f is differentiable almost every where. The derivative f' is measurable and

$$\int_a^b f'(x)dx \leq f(b) - f(a).$$

Or

(*b*) Let f be an integrable function on $[a, b]$. and suppose that $F(x) = F(a) + \int_a^x f(t)dt$. Then $F'(x) = f(x)$ for almost all x in $[a, b]$.

6. (*a*) Show that L^p spaces are complete.

Or

(*b*) A normed linear space X is complete if and only if every absolutely summable series is summable.